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THE PREPARATION OF THE TEACHER OF MATHEMATICS IN SECONDARY SCHOOLS.

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THE RHYTHM OF HISTORY

Of all the fascinating phases of the culture studies none has more to commend itself either to our æsthetic instincts or to our practical needs than the rhythm of history. The world's tastes of yesterday, its ideals, its methods of business, are discarded today to seeming oblivion, only to reappear tomorrow. History pulsates; the graphic arts flourish, they languish, they flourish again, and so on in endless rhythm; the tastes of the schools in one generation are not those of the next, but in due time reappear on the crest of the wave; religious zeal seems to have conquered the world in one century, only to be followed by a period of spiritual depression, and so throughout the range of human interests. But these pulsations, these waves of the human soul, are by no means alike; the great law of evolution decrees that each succeeding one shall have its own characteristics, even as those caused by a pebble in a pond are of increasing breadth and circuit.

Now the relation of mathematics to natural science is subject to this same law. With Thales it was close, with Pythagoras it was less so, and with the disciples of the latter it was still less; but with Plato it turns, and with Aristotle it becomes close again; with Euclid it relaxes, and with Archimedes it strengthens; and so on until modern times. See, for example, the calculus, devised as a tool for physical research; in the next generation more abstract; in the time of Laplace and Lagrange again chiefly an aid to natural science; in the next generation a subject of investigation abstract from the scientific application; and although in the hands of Lord Kelvin exactly what it was in the hands of Newton, in the average

American college course of today a dull, meaningless mass of theory with few applications, with little interest, and of value far less than it might easily possess. Hence it seems to me a very hopeful sign that, even as in the days of Archimedes, of Roger Bacon, and of Newton, the general sciences drew close to mathematics, so in this association an intimate relationship should be recognized.

THE UNION OF MATHEMATICS AND SCIENCE.

And yet, even with this hopeful sign comes the ever present danger, the thought that the crest of the wave will be followed by a depression. For when science gives herself completely to the embrace of cold and calculating mathematics she never fails to repent, and in her repentance she again seeks her own; and when mathematics yields too much to the warming influence of science he gives of his own virility for the time and ceases to progress. When, however, each supports the other in perfect union, neither forgetting its own proper mission, then we have an offspring of ideas that grow strong and help the world to progress. It is with the desire to see such a union that I welcome the consideration of this topic by an association of science teachers.

WHAT MAKES A MATHEMATICIAN.

This relation of science to mathematics (and be it understood that the word "science" is used for brevity in its popular sense, mathematics being even more of a science than the "sciences")—this relation of science to mathematics leads to the remark that the idea of what constitutes a mathematician, like all ideas, has had its own rhythmic progress. If you will take a small map of India and place your thumb on Bombay and your forefinger on Calcutta, and then will close them slightly and move them a little towards the top of the map, they will rest on or near the two greatest mathematical centers of ancient India, the finger on Patna where Aryabhata lived and labored fourteen centuries ago, and gave us the oldest complete Hindu mathematical treatise that has come down to us; and the thumb on Ujjein, where a century and a half later Brahmagupta made a mathematical Mecca for his countrymen. This sage of Ujjein has given us a definition of a mathematician, and one which we may well consider in thinking of the making of a teacher; for, primitive though it be, it is not without its suggestiveness. Brahmagupta says: "He who distinctly and severally knows addition and the rest of the twenty logistics, and the eight determinations, including measurement by shadow, is a

mathematician." (I might pause to inquire, in view of this definition, if there is a mathematician here present.) Now what does this mean? Brahmagupta's twenty "logistics" were: our four fundamental operations, squares and cubes, with their respective roots, five rules in fractions, the rules of 3, 5, 7, 9, 11, and barter; his eight "determinations" were mixture, progression, plane figure, excavation, stack, saw, mound, and shadow,—to use his own phraseology. Now the details of all this will not interest us particularly, but the quaint language may catch our attention sufficiently to have us remember that a mathematician is not merely a man who knows the theories of the subject, but one who is able to use those theories in a practical way. And this has generally been the case with the world's geatest leades in this field. Archimedes, for example, wanted higher mathematics that he might use it, and so he laid the foundations of the calculus nearly two thousand years before Leibnitz and Newton, building immediately upon the works of Kepler, Cavalieri, Fermat, and Descartes, created the science as we know it. And so these other men just named, and Euler, Laplace, Lagrange, Legendre, Monge, Gauss, Cauchy not to speak of Klein in our own day, men whom we look up to as among the world's greatest mathematicians, all knew the larger uses as well as the mere theories of this oldest of all sciences.

THE TEACHER VS. THE MATHEMATICIAN.

Now a teacher of mathematics stands somewhat apart from the mathematician himself. I know very few men of recognized standing as investigators in mathematics whom I would dream of recommending as teachers of the subject; neither do I know many who are today moulding the teaching of mathematics whom I would recommend for positions requiring original investigation in mathematics. And this is merely a corollary to a psychological proposition; if our brain cells have become so formed or so interrelated that we, at adult age, have interests abnormally along one line, it is exceedingly difficult for us to concentrate attention on another line. We express this in a homely way by telling the cobbler to stick to his last. So it is only fair to say that an ideal teacher, an exceedingly difficult person to find, while possessed of an all-round knowledge of mathematics and interested in seeing it develop, can not be expected greatly to assist this development by his own original discoveries. On the other hand, a man who is

doing really great work in mathematical investigation, and such men are also exceedingly difficult to find, while interested in seeing the subject well taught, can not be expected greatly to assist in advancing the educational thought by his own efforts. The great teacher and the great mathematician are equally rare; they are equally to be honored; but they are never combined in one individual. And between these extremes the law still holds; as a man leans to one side, his interests fail in the other.

DOUBTS AS TO PROFESSIONAL TRAINING.

Now this law is often denied, both by original investigators in education and by original investigators in mathematics. We have in this country a good many pretty fair teachers of mathematics who think that they are great mathematicians, and it may be well that they should think so, since it increases that all-round interest in the science which it is so necessary for them to possess; but in reality they are mere dust floating about in the gigantic machine. On the other hand we have a good many men who are contributing more or less to the progress of mathematics, and who think they are great teachers, when they are ignorant of even the primary principles of education, usually even doubting that such principles exist. The one who is interested primarily in teaching, sometimes displays his ignorance by openly expressing non-appreciation of the work that is being done in higher mathematics, and thus he shows a narrow mind that excites the pity of all who look fairly at the progress of the world. On the other hand, the one who is interested primarily in mathematical research sometimes displays his ignorance by openly expressing his disbelief of the work that is done in the professional training of a teacher, a training which would at one time, when his mind was receptive of something besides mere mathematical truth, have been of service to him. This antagonism to matters educational arises from the fact that there was for a long time a tradition among those who ranked themselves as scholars that the guild of teachers burned incense before some fetich called Method, without worshiping at the altar of true knowledge; that its members bowed before form without content, shadow without substance.

In the early days of normal schools, and long before the professional college was thought of, there may have been some justification for the charge. But with the rise of the standard normal school of today, with at least two years beyond a four-years' high-school course, usually combining advanced academic work

with the professional, and with the birth of higher professional colleges, and with the growth of special training in other professions, the tradition has lost its earlier dogmatic form. Two generations ago physicians often asserted that the knowledge required for their fraternity was best obtained in the office of a good practitioner, but today we require not only graduation from a good medical college, but even postgraduate work. The lawyer, the preacher, the nurse, the bookkeeper, and the teacher as well, all demand knowledge peculiar to their several professions, and in advance of that of their immediate predecessors.

GROWTH OF THE DEMAND FOR PROFESSIONALLY TRAINED TEACHERS.

These facts have been so generally recognized that there is a rapidly increasing demand for professionally trained teachers of mathematics, as of other subjects, in secondary schools and colleges. The time was when part of this demand was satisfied by graduates of academies and normal schools, but this time has passed. The smaller villages may still have to employ such teachers, and unless they will pay enough to get a good class of college graduates it is better that they should; but all of the more responsible positions now demand the college degree, and rightly so. In addition to this, a graduate professional year is coming to be required wherever it is possible to find candidates who have taken it, and the future will see the doctor's degree demanded, plus or together with this advanced professional training. Last year half a dozen of my graduate students secured positions in college faculties, a fair index of the value placed upon the professional work by the very institutions that recently were doubtful of the advantages of a course in education.

THE NATURE OF THE WORK OF MAKING A TEACHER OF MATHEMATICS.

This too long preliminary discussion brings me to the question as to the preparation demanded today for a satisfactory teacher of secondary mathematics. I say "today," because tomorrow will demand more, and I say "secondary," meaning to include high schools, normal schools, and the freshman year in college.

In answering the question, it is unnecessary to discuss the matter of personality. That is largely beyond the powers of the professional school. All that we who are training teachers can expect to do is this: other things being equal we can make what we sometimes hear called a "born teacher" much better able to do

good work, saving him from a multitude of blunders, and showing him the large questions which he should not allow to be obscured by a multitude of unimportant details.

First, what academic work does he need in mathematics? As a minimum he must have two years of college work, carrying him through the elements of the calculus. Any less than this would unfit him intelligently to handle either geometry or algebra. In addition to this my own students are practically required to take projective geometry because of its bearing upon the Euclidean science, and the advanced theory of equations because of the light which it throws on algebra. But most of my graduate students take much more than this, the theory of functions, differential equations, solid analytics, the theory of surfaces, and modern ideas in geometry being the courses from which selections are most frequently made. My recommendation is that every student shall take as much work as possible in pure mathematics, consistent with the other important demands upon his time. Of these demands one is that of applied mathematics. I am urging more and more strongly that our students should take courses in general applications (and we have recently established such a special course for teachers), in mechanics, and in astronomy. Any other plan would fail to give that balance between pure and applied mathematics which is necessary to the making of a teacher who would show the practical uses of the subjects which he treats.

Secondly, what professional courses does he need? In the first place he should know how mathematics has grown up. Ignorance of this fact would tend to make him unprogressive; he would look upon mathematics as a fixed science, undeveloping and undevelopable. This is the attitude of mind of the majority of teachers today. Against any proposed change they invoke the fetich of "mental discipline," believing, with an ignorance fostered by indolence of mind, that what has been good enough for the Past should be sufficient for the Present. Hence at least one year's work in the history of mathematics is demanded of all of my students, and a second year's work is advised, and I know of no better stimulus than this for the appreciation of a purely professional course. To know how elementary geometry has been crystallized in its present form, is almost a *sine qua non* to intelligently considering any question of the reform of the subject; to know the early geometric methods of solving equations is necessary if one would appreciate several improvements in teaching

which are now being advocated; and to know the early story of the calculus is equally important if we would rescue the subject from its present status. These are but a few illustrations of the innumerable lessons which history has for the teaching (not to speak of the developing) of the subject.

And finally, after the student knows the science, its history and its applications, at least one year of professional training in the teaching of mathematics is necessary. In this the successes and failures of the masters should be made known to him, encouraging him to do better work, and warning him against well-recognized dangers. He should get, what so few teachers have, and what mere academic instruction does not bring, a knowledge of a teacher's library, of the world's best textbooks, and of the literature which appeals to the pupil. There should also be brought before him the best methods of treating typical chapters, the possibility of improving the sequence, and the important question of the interrelationship of the branches of mathematics and of their contact with the other sciences and with the life about us. If we are to effect the reformation in algebra that has been effected in arithmetic, if we are to vitalize our applications of equations, if we are to modernize our geometry along the recent French lines, if we are to put trigonometry where it belongs and teach it as something to be used, and if we are to work out the various other problems of similar nature that are at this time pressing upon us for solution, it will be possible largely through the efforts and co-operation of that earnest body of workers which is constantly increasing by the advent of college graduates with a broad professional training. For such a body of workers the school has great need. Never has there been such a seething period in the educational world. The questions of the improvement of the teaching of mathematics are no longer local, no longer national; a reform in Italy is known at once in America, in New Zealand, and in that eastern Island Empire whose recent success in the field of battle and on the seas is paralleled by its advancement in education as well. To keep pace with other countries, to take what we can use and reject what is not suited to our needs, and to keep pace with the demands of science, of engineering, of commerce, of labor, and of finance—all this lays heavy demands upon the guild of mathematics teachers, and the proper training of apprentices who would enter that ancient and honorable order may well command the interest and the thoughtful co-operation of an association with the ideals this one so carefully guards.

AN IDEAL COURSE IN HIGH SCHOOL PHYSICS.

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While it seems hardly necessary to speak to science teachers about the reasons why Physics should be taught in the High Schools it may be well to mention some of them in order to give the proper background to the following discussion. If we have the same idea as to what we wish to accomplish by the course, it will be easier to derive ways and means to obtain the desired results. Let me say right here that while I see clearly the goal to be reached I must leave the practical working out of the method to more competent men and women, the high school teachers themselves. My paper will therefore be restricted to a discussion of the general, fundamental principles; we must agree upon these first before attempting a reform. That not all propositions made in this paper can at the present time actually be carried out, I know very well, and I have therefore called this outline an "ideal" course, but fully believe that when finally—and I hope it will be soon—adopted, it will meet the demands made upon a physics course, better than the one now usually found in the high schools.

Scientific training is now generally admitted to be an important part of any man's or woman's general education, and opinions differ only as to its relative value. The pedagogical importance of science courses lies principally in the following three facts:

(1) *It gives us a practical acquaintance with the world in which we live and a high appreciation of its grandeur.*

The second part of the sentence was added designedly. It is too often stated that scientific training deprives us of ideals and makes us unfeeling reasoning machines. What a libel on science! Could it be possible to become less of an admirer of this great universe and of its unchangeable laws, because we learn to understand some of the parts of this grand machinery and the way in which they are linked together? It is true, man loses some of his self-made halo; he is no more the all important selfish center around whom and whose doings all things are grouped, but becomes only a part of creation. I think that if science teaches us nothing else this is a sufficient reason for its being included as a culture study in the curriculum of every school.

(2) *It strengthens the power of observation.* We are not a nation of dreamers; we must grapple with the problems that arise

before us every day, and who is better prepared to do so than the one who has learned to observe clearly and who can distinguish the important features of each problem. Not only an open mind, but also an open, fearless eye is needed.

(3) *It gives a clear realization and appreciation of the connection between cause and effect.* The relative simplicity, the law and order, everything apparent in nature must necessarily tend towards the development of a logical mind. While in the action of mankind we may often search in vain for the underlying motives, while the study of languages requires in part a memorizing of a disconnected collection of rules without a guiding thought, science can be built up from a few fundamental principles more complicated ideas are deduced from the simpler ones, every phenomenon becomes of necessity the effect of well definied causes. Our confidence in nature and with it in our own actions, if they are in accord with the eternal laws is strengthened by science as it cannot be by any other study.

I have thus reiterated here in a few sentences three well known arguments in favor of science, teaching, not because I think we find at the present day many people who deny science a place as a culture study, but because I wish to emphasize its importance to *every* man or woman, no matter at what period of life they leave the school and because a lively realization of its importance is necessary for my first proposition which forms an integral part of the "ideal" scheme.

I believe that a general scientific course, descriptive in its nature, and concerned only with the exposition of facts should be given early enough in the school curriculum so as to reach every American boy and girl. Certainly we cannot hope to accomplish in such a course all that was mentioned before, but we can lay the foundation for future growth. One single science cannot give us the desired result, let them all be combined into one course; let the child learn to look around, teach him something of the heavenly bodies, of the history of the earth, of the life of animals and flowers, of the weather and certain frequently recurring fundamental chemical and physical phenomena. A course as this, aptly called a course in "phenomenology" should be given not later than in the Eighth grade and I am sure it will arouse, if properly taught, an interest which cannot be equaled by any other study. Some of the topics mentioned are taught already, but only as side shows or

in a very incomplete way. What we need is a systematic course given by teachers, properly prepared in all these subjects.

After this somewhat lengthy introduction which belongs however to the scheme, we come finally to the subject of the paper, the teaching of physics in the high school. Let me right here assure you that I have endeavored to look at the problem, not from a University teacher's point of view, but from that of a man who loves his science no matter where and with what object in view it is taught. The course in physics should not be treated as a possible requirement for entrance examination for this or that college, but as a preparation for life; as such it should have a distinct and honorable place mong the subjcts studied in the high school.

Recently we have heard a good deal about the lack of interest in our science, shown by the pupils and a demand for a complete reorganization of the work. This had led to the "new movement among physics teachers" tending to remedy the evil spoken of. I am much interested in this movement which is probably known to a large number of you, and I heartily agree with most of the suggestions set forth in the circulars of the committee.

It is recognized generally that the most serious charge made against the present method of teaching is, that too much emphasis is laid upon mathematical and abstract ideas and too little upon the practical applications. This is serious indeed, for it is only in its applications that physics becomes of value to the people in general. The teaching of our science has among others the following distinct aims:

(1) *It teaches those natural phenomena which make up the largest part of and have the greatest influence upon our physical life.* With other words, it has to enlarge in its specific field upon the subjects in an elementary manner in the course in phenomenology. Since physics should be taught before the third year in the high school the pupil can now learn to give up his egotistical, emotional attitude towards the world and do some impersonal thinking. Now a deeper understanding of natural phenomena and their dependence upon the fundamental laws can be expected, and some of the clearly defined relations between physical quantities established. Mathematics, without which physics is only an unreal useless shadow of a science, must be used, but it should be of the simplest kind. But the greatest emphasis upon the fact that we have here more than mathematics, which is only a tool. The different factors of an equation must never lose their concrete

physical meaning. Ideas of velocity, gravity, atmospheric pressure, temperature, quantity of heat, change of state, nature of sound, light and color should become clearly defined and their constant occurrence in daily life shown. Incidentally physics accomplishes another purpose which should always be kept in mind by the teacher.

(2) *It gives us the reasons for actions which are commonly said to be demanded by common sense.* While I am far from considering teaching of common sense and physics as identical, that desirable mental quality is surely strengthened by a properly conducted physics course. A man who has learned his physics—and let us add,—takes the time to think before he acts—will avoid many useless or foolish things. But we may go too far in this comparison. I have heard it asserted that a man who with his family left his house during the winter months without turning off the water in the cellar, did not know physics though he might possibly have been a university professor of this science in a university. Let us not draw such a conclusion about a man's knowledge, if he should not show common sense in spite of knowing better. Physics is not a sure antidote guaranteed to cure without fail absentmindedness, carelessness and who knows how many other human failings.

(3) A third and most characteristic advantage of a physics course is, that *it makes us acquainted with the important applications of physical laws to domestic comfort, means of communication and transportation, in* short with the world's work. Take for example the various heating appliances in modern houses, the electric bell, electric lighting, telephone and telegraph, optical and musical instruments, machinery of transportation all these are subjects which an educated person should understand. I expect to hear the objection that there is not time enough to treat these subjects in detail. Though I hope and feel sure, that some teachers find enough time I will admit the argument; but then we are unfortunately forced to omit a part of physics which has not yet proved to be of as much practical value. Let us sacrifice some for the greater good that can be accomplished by being a little less systematic. I believe one of the greatest faults of our present high school methods is that they are too much a thoughtless imitation of a university course which latter is given to maturer minds and would be incomplete without covering the whole field. A few important subjects mastered by the highschool pupil will do more

good than a hurried survey of too much ground. The Atwood machine which gives good results only in the hands of a skilled experimenter, the mathematical side of simple harmonic motion and polarized light may be omitted, not to speak of others. Only the fundamental notions of electrostatics should be presented, even if we lose some of our showiest experiments. I am sure that few college professors would demand a knowledge of these subjects for entrance examination. What we want is to be sure that a student has acquired a habit of thinking logically and to do his thinking for himself.

I do not wish to be understood for the complete abolishment of memorizing in physics. I have special reference to definitions. We have quite a number of technical terms which in ordinary life are sometimes used with a definite meaning, but often only to make a well sounding sentence. Take such words as acceleration, force, power, energy. I contend that a proper study of physics is impossible without a knowledge of the exact meaning of these fundamental concepts. They are to the average pupil almost like words in a foreign language and as such have to be learned, simply by memorizing their meaning. Against this absolutely necessary exactness is sinned more than in any other way. Of course, there are different ways of teaching definitions. As pure abstractions they become tedious; but you should lead up to each by well chosen examples of every day life and show the necessity of a critical attitude in the use of the words.

I do not agree at all with some of the reformers who say we should sacrifice exactness. The adolescent mind strives towards exactness. The transition stage from the instinctive, passive to the rational, active basis of life—to use Professor Starbuck's words—"characterized by clearness of definition, and is fully organized on the basis of logical order and sequence. During childhood the force of law and order has been largely external; but now the person must see it for himself, he must be the embodiment of law. The youth turns logician and proves everything and accepts that only which seems to possess a reason or for which he can construct one." Who has not noticed the passion for argumentation and the censoriousness of these young critics?

Adolescence is also characterized by the rapid development of the power of imagination. I like to stimulate the imaginative power of my students, *i. e.,* the power to see things and phenomena ahead, but that does not mean an abandon of exactness. I

fully agree with Professor Franklin when he says: "Many students and even teachers, of physics raise the objection that a rigorous presentation of the theoretical structure of physics is highly unsatisfactory and uninstructive and of course this is true if the physical facts themselves are lost to view. Many teachers prefer to discuss the "results" of science.—Really such a student should be treated honestly and placed under the instruction of Jules Verne where he need not trouble himself about foundations but may follow his teacher pleasantly to a care free trip to the moon.—There are too many people who fancy that they have an interest in the results of science and who, poor fools, invest in Keeley motors and Sea Gold Companies because, forsooth, the desired result is so clearly evident."

Unfettered fancy, quite a different thing from imagination, and the gloating over glittering generalities without a solid background, have too frequently been the ruin of an otherwise promising life.

Our science cannot be learned by book-reading alone, but must partly be taught by the laboratory method. The pupil wishes to make practical use of what he knows. I will add therefore as the last point the following:

(4) *Physics teaches not only accurate observation, but by training in measurements* (laboratory work) *shows the way to successful practical applications of its laws.* Physics is a science of measurement and the great advance in recent years has been due to a great extent to knowledge obtained by accurate measurement. A new theory without the backing of numerical data will have little hope of convincing a physicist of its superiority. No one can enter into the spirit of modern physics without a taste at least of what accurate measurements are. Though I surely do not wish to have a high school pupil discover new or old laws of nature, I believe strongly in quantitative exercises in the laboratory; a measurement should not be made for the measurement's sake alone, but always with some practical application in view. Do not use the vernier caliper in order to find out if a pin is round, but as a preparatory measurement of an object which later on is used in some practical experiment. Do not require measurements of physical quantities the pupil will never again meet in his life; do not attempt quantitative experiments in cases where the theory would be too far advanced. In such cases quantitative exercises will be fully sufficient. But in general qualitative experiments should be re-

stricted to the lecture room. I consider an experimnt, often performed, namely to connect a battery to an electric bell and make it ring, nothing but a pastime. Place in series a resistance, vary the intensity of the current, calculate the minimum current necessary to operate the bell; possibly show the effect of polarization and you will have an exercise which is not simply play, but gives us a good illustration of Ohm's law Are we not going a little too far in our attempt to reform, if following the advice of some educators we surrender unconditionally to the likes and dislikes of the pupils? and confound "easy" courses with such of real educative value? The latter is always hard enough to be an incentive to the earnest student, which the former can hardly claim, though it may suit the majority much better?

I cannot close without at least mentioning a point which to my mind is at the root of the whole trouble with our physics teaching in the high schools and that is the college preparation of the high school teachers. I wish to speak only of those who knowingly prepare themselves for it, not of those whom an ignorant school board makes physicists at short notice.

If one writer says that a man who has taken advanced courses in college has become unfit for the teaching of elementary physics I agree with him if by advanced courses are meant such as prepare the student to become an investigator, but I most assuredly differ from him if he means that no courses in advance of the general lecture course should be taken. I have still to learn of a case where a man who has taken the proper advanced courses was less successful, all other conditions being equal, to one who had less preparation The great trouble, at least in the middle West is that the future high school teacher does not get a sufficient training in the lines he is intended or expected to teach. Often it is considered sufficient to have taken the college course in general physics with the little laboratory work which goes with it. How can we expect a man thus prepared to be independent? Is not he just the one who will make an attempt to transfer the college course—the only one he is acquainted with to the high school, transplant it into a soil for which it is not fitted and never was intended? We hear of an "attempt to force the worst features of college instruction upon the secondary schools." I know that a large number of my collegues would deny most emphatically ever having made such an attempt. I return the charge by saying it is frequently the high school teacher who has forced himself by limiting

his preparation to a single course in college to adhere blindly to it. The general Physics course in college is not intended to be given to fuutre teachers alone, but also to general students, future investigators and frequently to professional students. The preparation of the physics teacher begins when this course has been finished. There should be one more year in laboratory work (not research) of which the second semester could be spent if we follow the wish expressed by many teachers with lecture table apparatus, the projection lantern and electric machinery. Further there should be a course in the history of physics, one in meteorology and, if time allows, a short course in instrument making. All these courses combined will count up to about twenty-two hours; truly a small amount in a total of 120 hours. It must be admitted that the colleges have in some cases not provided for such work. Let us work together for its introduction into the college curriculum. I hope one of the steps taken by the national commission is the definition of what constitutes a well prepared physics teacher in the high school.

In conclusion one more word about the teacher. Our science is making a marvelous progress all the time. While the physics teacher should not be an investigator, he should have advanced far enough to be able to keep up with this progress. How can he be expected to do so unless he has some time to devote to further study. His school duties should be lighter than those of persons who have to teach subjects whose presentation is in the main fixed in its nature.

This is then what I consider under existing circumstances (in Iowa) an "ideal" course and a teacher prepared to give it. I would unhesitatingly leave to him the pedagogical side of the subject. The high school teacher is the authority in this respect. I can only tell you what from my point of view is necessary to make physics one of the most important and inspiring studies in the high school.

TO UTILIZE SCRAPS OF PLATINUM.

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An interesting and valuable "inorganic preparation," and one which will use to good advantage the pieces of platinum foil and wire which accumulate more or less rapidly in all chemical laboratories is to change such scraps of platinum into pure hydroplatinic chloride, $H_2 Pt Cl_6$. In order to prepare this compound it is necessary to remove the two or three per cent of iridium which is nearly always in alloy with platinum.

Five or six grams of the platinum are carefully cleaned in the palm of the hand with moist sea sand and these dissolved in a porcelain evaporating dish on the water bath, with aqua regia. Successive portions of aqua regia must be added until all the metal is dissolved. The solution is evaporated nearly to dryness. A little distilled water is added, and the evaporation continued. It may be necessary to add ten or twelve successive portions of water until all odor of nitrogen peroxide is absent.

Water is added to the residue which is filtered; the filtrate is heated to boiling. Caustic soda is added to strong alkalinity, then a few drops of ethyl alcohol. It is next acidulated with hydrochloric acid and a solution of potassium chloride is added. After standing some hours, it is filtered. The precipitate is washed with a dilute solution of potassium chloride.

After thoroughly drying the precipitate, it is removed to an evaporating dish, and crushed to a fine powder. It is removed to a combustion tube of about 40 cm. in length. This is placed in a combustion furnace, and heated for half an hour with small flames, while a stream of dry hydrogen passes through to reduce to metallic platinum.

When cool, the substance is transferred to an evaporating dish, and crushed to a fine powder. It is washed several hundred times (1,000-1,200 times may be necessary), with hot water by decantation, until a few drops of the washings give no precipitate or cloudiness with silver nitrate. This metallic platinum is dried and weighed on the rough balance in order afterwards to make a five or ten per cent solution of the hydro-platinic chloride. It is transferred to an evaporating dish, dissolved in aqua regia, and evaporated while distilled water is added from time to time, until

no further odor of nitrogen peroxide is discernible. The residue is dissolved in water up to the required strength.

The experiment is not above the reach of the ordinary high school student, and is one which will keep the restless, energetic boy busy during his laboratory periods, for at least two or three weeks. The following are the equations that express the principal reactions involved:

$4HCl+4HNO_3=4Cl+4H_2O+4NO_2$

$Ir+4Cl=IrCl_4$

$2IrCl_4+8NaOH=2Ir(OH)_3+7NaCl+NaClO+H_2O.$

The few drops of alcohol decompose the sodium hypochlorite with the formation of acetaldehyde.

$NaClO+C_2H_6O=NaCl+CH_3, CHO+H_2O$

$Ir(OH)_3+3HCl=IrCl_3+3H_2O.$

The iridious chloride is not precipitated by potassium chloride while the hydro platinic chloride is precipitated.

$H_2PlCl_6+2KCl=K_2PtCl_6+2HCl$

$K_2Cl_2,PtCl_4+4H=Pt+2KCl+4HCl$

$Pt+2HCl+4Cl=H_2PtCl_6.$

COOLING BY EXPANSION AND WARMING BY COMPRESSION.

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The following method of showing cooling by expansion, and condensation of the water vapor of the air into a visible cloud of water particles, may be of interest to instructors of physiography. It is a method I have used with success for several years. The apparatus necessary is: 1. An air pump. 2. A bell jar. 3. A bottle with snug fitting cork, coated with vaseline. The bottle is corked and placed under the bell jar and the air is exhausted from the bell jar. The cork is pushed out of the bottle by the air inside. The sudden expansion causes cooling enough to condense the water vapor into a cloud which remains visible for a considerable time. Slow leakage of the air into the bell jar may produce warming by enough to re-evaporate the water. This warming by compression is made more striking if the bell jar is provided with a stop-cock by which the air may be admitted more rapidly and in a manner which is apparent to the class. The success of the experiment varies with the humidity of the air, but under the most unfavorable circumstances, it is never an entire failure. The size of bottle to be used and the force with which the cork should be pressed into it can easily be determined by trial. The cloud in the bottle may be made more clearly visible from the distance, of course, by providing it with a proper background.

BIOLOGY AS METHOD AND AS A SCIENCE IN SECONDARY SCHOOLS.

By Henry R. Linville,

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Not an uncommon criticism on the results of school as well as college teaching of biology is that our much-vaunted scientific method does not show any appreciable effect on the minds of those who devote themselves to biology for even a considerable length of time. I have heard it maintained by skeptics that, for example, botany and logic have nothing to do with one another, so obvious is it to them that a man may be a good botanist and not a logician. It would seem that a botanist or a zoölogist might carry through even a work of investigation arriving at apparently sound conclusions, without retaining thereafter scarcely a trace of the impress which the employment of the logical method of thinking should have left upon his mentality.

Another important criticism on the results of our teaching frequently made by those who are not biologists at all, is that the apparent subject-matter of an ordinary course in botany or zoölogy consists of a congeries of facts more or less closely related among themselves, but having no evident connection with the life of man. To these critics it seems that biologists as a class contribute little or nothing out of their store of knowledge to the solution of the manifold problems of human life. Do scientific biologists contribute no more to the progres of human society than, for example, the scientific philologists?

Before considering these two criticisms it seems to me sound that ultimately no branch of science or learning of any kind can stand in our educational programs, unless it is able to meet philosophical criticism from every source. The more we biologists stand together establishing a little realm of thinking, or lack of it, to ourselves, the greater will be the fall when it comes. When in these days of highly trained professional biologists, many of us sneer at the essays of sociologists in our field, we forget that many men who were not biologists primarily, from Aristotle to Spencer, have made substantial contributions to our store of knowledge and to our philosophy. And so, when thinkers along general lines say that biology as it is taught does not train to thinking, it may pay us to halt and consider the matter.

There is a large class of biologists that the question in its large relations does not touch at all. Most of the colleges support biologists who have no particular interest in the relation of biology to any other matter. Those of us who are on the inside know that a high percentage of the work published as researches is done by men whose chief ability lies in the direction of discovering facts that others have been too busy to look after themselves. The most ordinary power will enable them to tack these facts on to theories already formulated, and fairly established. As to thinking beyond the reaches of what is called "common sense," there may have been none. Another group of biologists content with their ability to break out new lines of scientific endeavor, have scant sympathy with movements which have for their purpose the use of the subject-matter of biology for general educational purposes. But whether he will or no, the days of the pure scientist are drawing to a close. The leader in pure science cannot be a leader of the people unless he does something to relate his work to human progress.

If we concede that human progress in all its phases is the most important thing that can take place on earth, then it becomes appropriate to ask if any tangible force can be named which has been instrumental in attaining the end toward which we are moving. It is just as obvious that *the ideas of men* have given the impetus for all advances in civilization, as it is that the thinking power of the brain has produced those ideas. In the conflict of ideas worked out by the brain of all the races, those ideas which for some reason an appeal have made to the developing conscience of mankind, have gained the ascendency. The strongest ideas held by the several cotemporaneous and successive civilizations have existed as the result of the collective thinking of all the brains, although the expression of the ideas we may rightly credit to those leaders of thought who were able to formulate them. Now, we are accustomed to measure the civilization of the various nations by several criteria, such as intellectual, aesthetic, moral, social, and industrial development. The respect with which mankind regards development in any of these lines depends in a certain measure on his own mental environment, but even in our own country whose industrial development leads all others the elemental necessity of the ability to think must appeal to everyone.

I believe no teacher would question the statement that much if not most of our teaching in the schools consists in disseminating

information. Superintendents of instruction as a class fight against the introduction of any element, called scientific, that is supposed to render difficult the getting of information by the children. It is true that some subjects are supposed to provide mental drill, but what kind of mental drill do we mean? Most apparently the ability to remember. The proof of it is to be observed in our examination system, which for the most part is a test of the memory, and seldom of the ability to think. So insistent are the controlling boards becoming, that a reproduction of the marvelous memories of educated Chinese, would seem to be the logical outcome of Western education. If this is to be the result, many of us will feel in entire sympathy with the expression of Themistocles, quoted by Matthew Arnold "Teach me rather to forget!" As Matthew Arnold continues, "The sarcasm well criticises the fatal want of proportion between what we put into our minds and their real needs and powers."

For some years, physiologists have been trying to discover what the "real needs and powers" of man are in regard to his food. Without doubt, a similar desire is at the bottom of the most valuable studies of our educational method. It seems to me that from the point of view of the good of all the people, the most basic need is the development of an organic power to think. I believe it could be demonstrated to those who have the power to think abstractly, that all of the injustice that is included in the phrase "Man's inhumanity to man," could be stopped if the people could be stirred to break from their dependence on hide-bound tradition and authority. If real self-reliance in our thinking is a primal need and of great value; and if our heritage of information that the books contain may be stored in imperishable buildings, rather than in perishable brains, then the sooner we enter upon a new era, the better.

There is a general impression among teachers that children do not begin to reason before they reach the high school age. And in the high school we hear the opinion that certain subjects should be taught for the information they contain, the presumption being that no thinking—only remembering—will be required. It is true that those children who enter the high school today are not, as a rule, able to think abstractly. It is also true that those parents and teachers who treat the minds of their children with respect, are able to see intellectual growth through the habit of leading them to understand the reason of things. We have never heard of any

scientific study which proves that children develop the reasoning faculty at a certain period, as they develop permanent teeth. It is rather more likely that people have repressed the natural reasoning faculty in children by requiring them to believe as they believed,—because some dominant personality said so.

Much as I believe in biology as an instrument of good in educating the people, I regard the dissemination of information concerning plants and animals as the least important of its uses; but for the most part, that is all we do. We fill up with all the pupils can hold, mostly because we do not know what else to do. We try to make the subject interesting, because we want them to be pleased with it. We sometimes avoid the hard places for fear the interest will lag. We pay tribute to the little gods and goddesses by giving out predigested mental food, but we have *made* them mental dyspeptics.—They are not that way naturally.

If we as teachers were not carried along by the tremendous impetus which the present system of education has gained, much as the individual pedestrian in Broadway is rushed along by the crowd behind him, more of us would have time to at least stop and think whither it is all leading. If such an opportunity were offered, I believe we might agree that one desirable object would be that we should have such a handling of subject-matter as would lead the children to consider the facts they learn in relation to other congeries of facts:—in other words, to interpret nature. For example, the idea of a tree would not be a dozen or more kinds of trees, and nothing more, but the tree as an organic thing would be explained or interpreted through the consideration of the seedling, the soil and water, and the air. At the other end of the story the meaning of the tree would be interpreted in terms of its relations to other plants whose existence is interfered with or favord by it; in relation to animals that use it for food or protection; in terms of the water supply for cities; and in terms of material for the use of man. This is not a new idea. We all know how poor an imitation of a scientist is the person who "knows all the plants," but I doubt if we keep before us the importance of the thinking idea as training in method. While knowing the dozen or more species of trees on the mountain may be but a feat of the memory, the knowledge of one tree or plant of other kind as an organism must come through the serious consideration of the relation of things. This process requires continual thinking. It is as much the logical process as anything can be, when in experi-

mentation to determine the relation of moisture, heat, air and light to germination and growth, the pupil is encouraged to analyze, eliminate, control factors, and to reason from results. The method of learning the principles of biology by experimentation *is* the scientific method. Those who criticise our work because it does not show in its followers the discriminating habit of the logical mind are in effect attacking our practice of verification. I have had the opportunity lately of examining the laboratory questions used in an elementary course in zoology in one of our foremost universities. The questions were nothing but statements, which the students are asked to verify. The students have no opportunity to discover for themselves the facts in regard to the form and activities of the Paramœium, through the exercise of their powers of observation, mental analysis, elimination, and judgment. On the contrary, they are being robbed of a splendid opportunity to develop those powers, and to gain through their exercise intellectual pleasure of high quality.

I maintain that the consideration of the logical relations of the science of biology, and the employment of the logical method of thinking in full or abbreviated form in learning the science, are of more than incidental importance. If there is any value at all in education, it comes through the additional power gained to the individual and to society. If civilization advances through the power of great ideas evolved by growing society and formulated by our leaders, then whatever help the education of the schools can lend to the evolution of ideas, society will be by that much the gainer. The greatest use that I could wish for the science of biology is that the practice of its method should promote the habit of thinking in men, and be of indirect use in suggesting, receiving and judging ideas of all sorts developed in the progress of the race.

There is a distinction to be made between the method of a science and the substance of the science in relation to the bearing that both have on education. The method is a logical system which may be abstracted from all relation to subject-matter, and serve an important intellectual end in developing the power of independent thinking. The science of biology must have educational value beyond its value as method; otherwise, it would be difficult to justify its inclusion in educational programs.

The unified body of knowledge which we sometimes designate by the term "The Science of Living Things" comes into the closest imaginable relation with ourselves as men. Some one has bor-

rowed from biology an idea and applied it in this form: "Education is adaptation to environment." I think that it cannot be doubted that the scientific understanding of organic environment by its comprehensiveness, its fullness and its characteristic of exhibiting things in their natural inter-relation tends to give the student the knowledge whereby he can see himself in a setting, so to speak. Instead of thinking of himself as an isolated creature, or at most, his race as an isolated race, he is educated to think of his race in relation to all organic factors that affect the race. Whether the student formulates his feeling or not, his intellectual life has the added balance and sureness which is evident in the thing we call power.

It is true that the young student of biology as he has been trained in the past could not attain to the largeness of view at which I have hinted, in the course of a few brief weeks, for the reason that the teaching of the science is yet in the experimental stage. Undoubtedly, we waste a great deal of time in busying ourselves and our students with looking at things without thinking much about them, and as the volume of known facts increases, the impossibility of passing them all in review becomes more and more apparent. We shall be driven to the necessity of concentrating our pedagogical energy, first on devising a scheme by which the method of biology shall be more immediately effective, and then to the task of so organizing the typical facts that the pinciples of the science may be got hold of in much less time than is now taken by the special student of biology.

It is apparent that before the knowledge of biology as a science can become general, we must save in time and we must also have more time. The need of this knowledge has never seemed so imperative as now. The imminent social revolution in the civlized world is being hastened by the ideas of the new science of sociology. Practically none of the professional sociologists have a working knowledge of biology, yet they are constantly basing their theories on biological phenomena which they know little about. Granting that the object of the sociologists is the betterment of the human race, their lack of ideal power through ignorance of the application of biological theories is all the more unfortunate. The sociologists realize their lack of efficiency, but they cannot get much work done as sociologists, if they are obliged to spend years as students of biology. I think the only way to help them and the people generally is by the system of teaching that

I have suggested, and by closer-knit and more effective organization of the subject-matter for presentation.

When the bearing of evolution, the most important principle of biology, on human affairs is fully comprehended by educated men, and the influence of the selective factor in hastening the process of evolution is realized, we may hope to see curricula contain a maximum of subjects that relate to human progress and a minimum of those that stand in the relation of organs that mark the progress of evolution, but hold slight promise of usefulness for the future. The process of natural selection, and even of intelligent selection, involves the retention of undesired characters. This fact helps us to understand how it is that certain unused languages are still retained in our courses of study, but it offers no excuse for giving more time to one of those languages than we do to any other subject.

Very great stress continues to be laid on the importance of Latin and other "humanities" for their mental discipline and culture-value. The implication has been that science does not hold as much mental discipline, and of course does not tend to culture, being materialistic and not idealistic. It is not apparent, however, that the study of Latin yields training much above that which a student would gain from the performance of any series of well-ordered tasks. He is taught to remember (if memory can be taught), and he is taught to be careful, but I cannot recall any bit of thinking which I did in the study of Latin that has been of service to me in any other relation of life. It is true there is said to be a logic of grammar, but is the logic of Latin grammar essentially different from the logic of English grammar, and is that what the defenders of Latin spend four years in teaching in secondary schools?

The further claim of culture-value in Latin and the "humanities" has lost much of its force since some of those who believe in it have begun to include the sciences in the humanities. But there is hardly any question that most of those teachers would claim greater cultural value for the ancient languages as against the modern sciences. The term culture as at present employed is so inexact a symbol that it is impossible to measure values with reference to it. If culture means the education the scholars got when little but Latin was taught, then I think we have done with it. If it means the state of enlightment which some regard as ideal for civilized men,—enlightment with reference to what the world

is, and to man in his multifarious relations, then not only is knowledge of the science necessary, but knowledge of history, literature and art also. It is evident that only through such enlightment are men put in touch with their environment. When they are in touch with their environment they have the feeling of acquaintance and power which enables them to be factors in the further extension of human progress.

It is probable that to some people culture means a sort of delicate refinement of knowledge in the line of classical literature and art, extending in some cases to modern literature and art. Of course we understand that the professional ancestors of these scholars were the first to own the word, and for a long time they can make it mean what they please.

I have discussed the method of biology, secondly the bearing of the science on the individual in his relation to society, and third the bearing of science in general on culture. I shall now say what I can on the aesthetic value of biology.

It has not been many years since the artists generally would say that it was impossible to formulate the reason for considering one picture better than another. The conviction regarding superiority seemed to rest in the emotions, or possibly in the artistic judgment, which appeared to be a different thing from scientific judgment. Now however, we find many teachers of art who believe that the formulation of principles can extend much further than drawing, and that the principles of composition and color-relation can be so refined and scientific as to get a tangible basis for a given aesthetic feeling. In biology, within the past ten years much has been written to the effect that in teaching nature-study our object should be to stimulate the love of nature, and to help to the appreciation of its beauty. I think that in proportion as we do this, we shall be accomplishing a splendid work, but the end must not be considered as an end possible immediately, for just as the artist must study anatomy and the principles of composition, so the student of biology must have a good store of facts and meanings. Moreover, in our present state of progress our talk about "the love of the beautiful in nature" is composed mostly of empty sounds. The phrase "going back to nature" expresses little more than an economic and social revolt against life in the cities. Children who are taken to the country every summer do not necessarily learn the beauty of nature, any more than do the country children who stay there all the time. And city and coun-

try children are not likely to agree on the beauty of a given place or object unless there is something in the education of both that leads them to see what it is that makes things beautiful. In other words, we must get at the principles of beauty. The education of the present day farmer is responsible for an expression I once heard when a splendid pine tree was rent by lightning, "What a shame, that tree was worth five dollars." To our botanist the tree would have been *Pinus strobus*. For the artist only would it have possessed the quality of beauty, in the sweep of its long dark branches against the sky.

The modern artists have borrowed from us the idea of basing their art on scientific principles, and thereby they are able to teach those who are not artists by nature. Through the influence of the Japanese, they are also making more and more use of animal and plant subjects in their decorative art. Can we as biologists hope ever to teach the beautiful unless we form an alliance with teachers of art? The idea that zoölogy is associated with ugliness is not altogether unfounded. We must seek to portray animals and plants in their *beautiful* as well as in their *accurate* relations.

In the relation I have suggested for artists and biologists, we get a glimpse of the possible benefit of co-operation of all branches of learning. Doubtless there are uses for all the subjects now taught in the schools. If they are to be made effective, they must be taught in co-operation in order to attain an end higher than any subject could reach unaided. In establishing closer relation between subjects, I would maintain that we must employ some criterion for estimating the importance of the various lines of study, and that the criterion be the value of the subject for contributing to human progress.

The criticisms that will continue to be made against biology as a factor in education while the teaching of the science is in the experimental stage, should stimulate us to search for the reasons for our professional existence. The philosophical considerations which have to do with biology and education are of course more fundamental than the expediencies of secondary school courses of study. Although the decisions of the hour may seem to help or hinder the establishment of biology and other sciences in the schools, they should have but slight influence upon a future ideal system of interrelated knowledge for the education of the people.

A VISIT TO A SUGAR REFINERY.*

By Harold Bisbee,

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The factory visited by the Association is a huge, brick building, located on Granite Street, South Boston. The raw sugar, which is the starting point of the refining process, comes chiefly from Cuba, Porto Rico and Java. This sugar consists of rather coarse, brownish crystals, slightly sticky, and of a sweet, agreeable taste. In the islands above mentioned the cane has been crushed, the juice crystallized, and the crystals separated from the molasses by centrifugal machines. It must be carefully borne in mind that the process to be described is simply a refining process; its scope is the change from these brown crystals of raw sugar to the pure white granulated sugar.

First, the raw sugar is made into a paste with the cheapest liquid obtainable. In practice, various by-product liquors are used for this purpose. The paste is then whirled around in centrifugal machines whose sides consist of fine mesh but strongly made metal screening. Through the innumerable holes the liquid is ejected, carrying with it a little dissolved sugar and a considerable amount of gums, resins and dirt impurity. The liquid is dark red in color, syrupy, and is a true molasses. It is not sold as such, however, but is mixed with a fresh lot of raw sugar to make another magma for centrifugal treatment, and so on, until the limit of its absorbing capacity is reached. For its sugar content the rich molasses liquor is then subjected to a treatment which will be presently described.

The sugar crystals which remain in the centrifugal are considerably lighter in color. This first step in the refining process has cleaned the exterior of the crystals surprisingly well. The interior must now be purified; and solution is the natural and necessary antecedent to this.

In the "melting pans" the sugar is now treated with hot water. The sugar "melts" or dissolves to form a light brown solution of a specific gravity of 29-30 Bé.

The defficating process, or the "blow-ups," is next in order. Here the sugar solution is heated continuously for several hours at a temperature ranging from 175 degrees F., at the beginning to

*This article is taken by permission from the report of the 26th meeting of the New England Association of Chemistry Teachers.—Ed.

185 degrees F. at the end until the gums and resins present have coagulated thoroughly. A test portion determines the completion of the process by a well-defined settling of the coagulated material.

The suspension must now be filtered. Cloth bags, about a foot in diameter by six or seven feet in length, are wrinkled up within an outer bag or "sheath" of about the same length, but of only half the diameter. Hundreds of these double filtering bags are screwed on to pipes from the "blow-ups." As they hang vertically down, close together, there is a certain resemblance to a vast series of organ pipes. The coagulated liquor, percolating through the "plaited" filter and the outer protective sheath, leaves behind the gums, resins and dirt, and issues forth as a light-yellow, surprisingly clear filtrate.

The molasses liquor referred to in a preceding paragraph, which has become charged with sugar and impurities by repeated washing of raw sugar, is usually defficated and filtered immediately after the purified sugar liquid has ben treated. The filter, although slightly soiled and clogged by its first operation, may still be used to filter the molasses liquor, which is almost black with coagulated material. The residue, which is obtained as a fine, brownish-black mud-cake after the filtering bags have been washed out and the washings forced throught a filter-press, is at present a waste product of the process. The filtrates, first and second, are kept carefully separate. and are pumped, in turn, into the bone-back filters.

These are huge cylinders, ten feet in diameter and twenty-four feet deep, filled to within six inches of the top with bone-black. As the solution works its way slowly downwards through the bone-black it becomes nearly or quite de-colorized in proportion as there is little or much coloring material to be removed. The filteration occupies twenty-four hours. After continuous action for two or three days, the bone-black becomes clogged, and must be re-vivified by ignition in retorts. In long rows of test-tubes are kept samples of the runnings from the bone-black filters. These runnings show a complete gradation from a clear, colorless solution to one of a dark amber color. They are classified by color; the colored liquors are filtered through bone-black again and, if necessary, a third time, till the color has been completely removed. The clear liquors are now ready for crystallization in the vacuum pans.

The vacuum pans consist of large closed kettles, about ten feet high by six in diameter, having a capacity of one hundred barrels of solution each. Pumps reduce the pressure to about two inches of mercury, then steam is conducted in pipes through the solution, raising the temperature to 160-170 degrees F. Under these conditions the liquid boils violently, without decomposition. When a certain concentration is reached crystallization begins. Now the process is watched with extreme care. Samples are removed in rapid succession and allowed to crystallize on a glass plate in the air. When the crystallization takes place rapidly enough to give the desired smallness of grain, the entire pan contents are run into large vats to cool and to crystallize.

Centrifugals now whirl out the mother-liquor, and gather a smooth, cylindrical coating of sugar crystals on their sides. This coating is washed with a calculated amount of water, and the whiteness is still further increased by a touch of bluing to complement the residual yellow tinge from the mother-liquor. All this is done while the crystals are whirling around in their cages (21 inches deep by 36 inches diameter) at the rate of from 600 to 800 revolutions a minute.

Out from the bottom of the centrifugals the sugar is pushed into a long, inclined trough within which there rotates slowly a longitudinal iron shaft with a screw blade. The moist sugar is gradually "wormed" along and thereby partially dried, preparatory to its entering the "granulators."

In the granulators the drying process is completed, and the crystals sorted out into the different sized grains. A large cylinder, six or seven feet high and thirty or forty feet long, rotates slowly about its longitudinal axis, which is inclined at a small angle, sloping forward and downward. Within the granulator are many cleats projecting a foot or more from the shell towards the centre. A closed central steam pipe furnishes heat for the process. The sugar is shoveled in at the upper end of the granulator, and little by little works down the length of the barrel. Meanwhile it is being tossed about in all directions by the cleats. and, in consequence of the complete surface exposure, becomes thoroughly dried. A peep into one of these granulators reminds one considerably of a winter landscape, with heaps of sugar lying everywhere in place of snow.

The thoroughly dried sugar is now bolted through various-mesh sieves to give uniformity of grain, and is then bagged or

barreled for sale. Weighed quantities of sugar are shot into bags, and the bags sewed together on machines with surprising celerity; four or five seconds only per bag are required. Barreling is an almost equally interesting process.

The mother-liquors from the vacuum pans are reboiled and recrystalized until they can be no longer profitably worked for their saccharose content. They are then filtered and sold, chiefly in European markets, as "barrel syrup," Barrel syrup contains 26-35 per cent glucose, and 30-34 per cent saccharose. Its chief uses are in making confectionary, table syrups, and patent medicines.

The total output of pure white, granulated sugar of the Standard Sugar Refinery of Boston is 1, 750,000 pounds a day.

THE TEACHING OF GEOGRAPHY.

By Professor Alex. Darroch.

The older methods of teaching Geography, which I am sorry to say in some cases still prevail, crammed the memory of the child with a host of more or less uninteresting facts, and did nothing either to feed or to cultivate the imagination, or to train the reason of the pupil. This is true whether the method relied on be mere book knowledge, or whether the teaching be supplemented by the use of the ordinary maps. The latter method was and is certainly an improvement on the former, but yet mere map understanding is not sufficient for the complete comprehension of geographical facts and their interrelations. Much less is it so if we endeavor, as we ought, to make our pupils realize that the geographical facts of a district condition, and to some extent are conditioned, by the more important natural and social phenomena whose exact study belongs to other but related sciences. For it is only by keeping this latter point of view ever in mind, that our geographical teaching can be made real and living to the child. Again it is being slowly realized that any kind of geographical teaching which trains the reason of the pupil to the perception of the causal interrelation between physical fact and physical fact is the best medium by which to introduce the youthful mind to the study of the Natural Sciences, and hence it has been well said that Geography teaching may be made the gateway to the teaching

of Science, for the subject of Geography, of all school subjects, if rightly taught, may enable a child to realize that the world in which he lives is not a mere sum of facts, but an interrelated system, a cosmos in which fact is related to fact and conditions and is conditioned by the nature of the whole. In this connection I may for a moment allude to an opinion held strongly by many educationalists of the present day. They contend that the exact sciences, such as Physics and Chemistry, should be preceded by a course of study having for its object the training of the pupil to perceive the larger and more obvious casual interrelations existing between the various parts of the Universe; and they further maintain that if we begin to endeavor to lead a young pupil to select out and to endeavor to understand the more minute and less obvious casual interrelations, before he has undergone a training in the discrimination of the first named set of relations, then our method is educationally erroneous and is one not likely to produce the best results in training the pupil in the methods of Science and in the formation of the so-called scientific habit of mind. For in the study of Physical Geography and the allied branches of knowledge, the facts to be observed and the interrelations to be discerned are easily separated out from the whole of which they form a part, and can be readily obvious to minds incapable, through immaturity, of understanding the more abstract and intricate system of causes with which sciences such as Chemistry and Physics deal. However that may be, all are agreed that the study of Geography in the widest acceptance of the term is one of the best instruments for extending the horizon of the pupil, for the cultivating of his imagination, and for training him to reason and to realize the Universe as a system. But a knowledge of Geography is not merely valuable as a preliminary to the more thorough and the more accurate study of the Natural Sciences, it is in many cases necessary for the right understanding of historical facts, and for the thorough comprehension of the history of our own and other countries. A moment's reflection will enable anyone to realize that geographical conditions have largely determined historical change in the past, and that the historical conditions existing at the present can be made completely intelligible only through the prior understanding of the geographical relations. Again, and above all a full knowledge of geographical fact is nec-

essary for the right understanding of the economic and commercial conditions of our own and other countries. For the localization of a people and the distribution of their industries within any one country and throughout the world generally can be made thoroughly intelligible only when our geography teaching has clearly realized this as its ultimate aim, and when the knowledge has been imparted to the pupil according to a sound method. As the final result of our teaching of geography, we should have made our pupils realize not merely that a certain city or town is placed here or there, but why this is so; not merely that it has such and such industries, but why these industries are located here and not elsewhere; and lastly, they must understand the use and function the particular city plays in the economic and social life of the nation and of the world generally. Only in so far as we have done this can we be said to have taught the subject at all. Further, while the economic and commercial aspect of this subject is so important for a country such as ours, from the narrow utilitarian point of view, that the knowledge and training is necessary for all those who intend in after life to enter upon a commercial career, yet it is also important from the fact that it is only by a method which endeavors to attain the end of showing the value of Geography for the understanding and interpretation of the economic and social life of a people that we can really educate the child and train his reason.

The best method of imparting geographical knowledge is to go direct to nature and learn from direct observation the causal interaction of a physical fact, and how the natural and physical features of a country or district determine the social and economic life of the people. This was the method of Rousseau; still more was it the method of that much greater educationalist, Pestalozzi, in the teaching of Geography. The cardinal principle of the educational method of Pestalozzi was that the symbol should never be given to the child until he had comprehended the thing signified by the symbol, and we are told that once when his own child used a term without a knowledge of the thing signified by it, he wept from sorrow at finding such an example of educational depravity in his own offspring. In his teaching of Geography Pestalozzi carried out thoroughly the maxim of presenting the symbol only after the thing itself was comprehended. In the once famous school at Burgdorf, the children along with

their teacher explored the country in the neighborhood of the school, collecting clay in baskets from the river banks, and on their return modeled day by day what they had learned of the physical nature of their environment and of the interconnection of physical feature to physical feature. Only after this had been thoroughly comprehended from the clay model was a flat map introduced and the relation of model to map explained. In our present-day teaching of Geography this method of going direct to nature should be employed wherever possible, and especially in the earlier stages, and any other method must be judged good or bad according as it approximates or falls short of the direct method.

But under the altered conditions of our time and especially in our large towns, the method of going direct to nature can be followed only to a limited extent, and, further, the method has limitations in itself which make its consistent use inapplicable under existing conditions, for it is obvious that the direct method is always limited by the narrow range of the child's environment. Again, the best method of teaching here, as in so many other subjects, cannot be employed in its entirety even if it were possible on account of the limited time in which the knowledge must be imparted. Hence the direct method must be supplemented by others, and all these subsidary methods must have for their aim the endeavor to make Geography a real, a concrete, and a living subject of instruction. If the Geography teaching of our schools has only for its results the memorizing of lists of names extracted from a book or gleaned from the unintelligent and premature use of a map, then whatever we may be doing we are not educating the child. Nay, more, by such methods we not only kill the child's present interest in the subject, but also fail to foster any future or later interest; and in many cases such teaching tends to make the child grow up dull, stupid, and unimaginative, and with reason dormant about the world in which he lives.

But we must also remember that the mere picturing of a district or of a country is not sufficient; the child at a later stage must be trained to employ his reason in the perception of the causal interrelations at work.

But if we limited our teaching of Geography to the mere understanding of the child's own district and his own country;

if our only aim were to make him realize how many factors have to be taken into account in the thorough interpretation of any one region, then we should not attain the highest results, and we should fail to realize all that we should endeavor to do in our Geography teaching. In addition to this we must further endeavor to lead our pupils to a knowledge and understanding —imperfect it may be—of the world as a whole and of the relation, position, and place of his own country to the whole.

It is not sufficient that we should know how to lead our pupil to understand and interpret his immediate physical environment, nor is it sufficient to know how to make him best realize the place and function of his own country on this terrestrial globe. We must further endeavor to make him comprehend the nature of the Universe as a whole and how this planet on which he lives is related to the solar system of which it froms a part and to other systems which the starry heavens reveal to our gaze. We may thus lead him to understand the vastness. the complexity, the grandeur, and the mystery of this Universe in which man at times seems to play but a poor and sorry part. —*Extracted from the Scottish Geographical Magazine, September, 1906.*

THE AGRICULTURAL HIGH SCHOOL.

By H. H. Lyon,

Bainbridge, N. Y.

Although it is true that we have but three leading types of high schools, "literary," "commercial," and "manual training," there is still a fourth that is attracting some attention, and which is likely to come in for a somewhat general consideration. I refer to the agricultural high school. The agricultural school can hardly be classed with the manual training school, for it has so many distinct features. Like those schools, however, it is not to be considered industrial, but chiefly educational. A prominent representative of the Educational Department of one of our great states said recently in a public address that "the scholarship of agriculture is a scholarship that stands with that of Greek or Latin." It is undoubtedly true, however, as was remarked by an official of the Agricultural Department of that same state only the day before the utterance just quoted, that,

"in the estimation of the so-called scholarly world, the agricultural college graduate does not rank with the graduate of the literary college." And he added, "To my mind, this is wrong."

Whatever may be regarded as the truth respecting the scholarship of the two college courses mentioned, the fact remains that we have the two courses, both requiring, practically, the same degree of preparation, and the same length of time for graduation. The agricultural high school is also an accomplished fact, with as complete a curriculum as the literary school, and requiring as highly trained a teacher, at least, in point of time spent.

Various state educational departments have been active in some degree in promoting agricultural high schools. In New England, one state, New Hampshire, seems to stand out more prominently than the rest in this respect. In those schools an agricultural college graduate is in charge. The schools are not large and the principal has immediate charge of the work, not only of the agricultural curriculum, but of the literary and commercial as well, all three courses being taught by competent teachers.

English is taught in every year in each course, algebra the first year and geometry the second. The agricultural course has a year of botany, one year of zoölogy and physiology, one of physics, and one of chemistry. There is the equivalent of one year in agronomy, rural engineering and horticulture, a half year in plant pathology and another half in a study of insects and insecticides. There are four credits, out of a possible five for a year in one study, in animal industry and dairying, and two credits for a study of farm management. Two years are given to the study of French.

THE TEACHER OF MATHEMATICS.*

By Henry L. Coar, Ph.D.
Marietta, Ohio.

One of the marked advances in the educational field in recent years is the recognition of the advantage of having teachers in charge of the various departments of our high schools, who have had some special training in their respective branches and who know a good deal more of the subject than they are expected to teach. This is particularly marked in the case of the so-called natural sciences. For some reason, however, this advantage has not been as much recognized in the field of mathematics, and we still find school authorities who seem to labor under the impression that anyone who has studied any algebra, is perfectly capable of teaching the subject, if he is capable of teaching at all. In the case of the higher institutions of learning much greater demands are being made of the teacher than were made even a few years ago, and a man must have had considerable training in mathematics to become the head of such a department, even in our smaller colleges. One reason for this is that the study of mathematics is being pursued by an ever increasing number of young men, and as a result, there are usually several well prepared candidates for every vacancy of this kind. For similar positions in the high school, however, we do not find the same number of similarly prepared candidates.

In order that the last statement may not be misunderstood, it is necessary to define the status of a well prepared teacher of mathematics for the high school. The question has been more or less discussed by mathematicians, and they now seem fairly well agreed as to what shall constitute a reasonable minimum of special training for a man to take charge of the department of mathematics in a high school. This minimum embraces the following subjects:

College Algebra, Trigonometry, Analytic Geometry, Calculus, Advanced Algebra (Theory of Equations and Determinants), History and Pedagogy of Mathematics, Physics (i. e., some applied work), Modern language, preferably German.

This minimum requirement may seem somewhat high, but

*Read before the Mathematics Section of the Central Association of Science and Mathematics Teachers on December 1, 1906, at the University of Chicago.

it is fairly obvious that a teacher cannot have that broad mathematical outlook which may well be demanded of him, unless he has covered about this amount of work. That a teacher needs such a mathematical outlook is almost axiomatic. Let us, however, stop for a few moments and investigate why he needs such an outlook and how the above mathematical knowledge will help him to secure it.

One of the aims of the teacher is to familiarize the student with the fundamental facts of elementary mathematics, and in many cases, this seems the only aim the teacher has in mind. How is he to decide what these fundamental facts are? Too frequently he is satisfied to take the word of the author for this and to consider most of the material in the book as fundamental. This is a mistake. The teacher himself should be able to make a decision as to what is fundamental, and the only way in which he can do so is to have pursued work in advance of his teaching and to have thus recognized from personal experience what parts of algebra and geometry enter most constantly into both pure and applied mathematics. He will then be able to emphasize these facts properly and to show his students by illustrations why these parts are so necessary. For both teacher and student to know why the subject is studied, must prove helpful and stimulating. In algebra, for instance, and I include here both elementary algebra and the so-called college algebra, we find the students almost universally in a sad state of mind. They grope about blindly, juggle with the symbols, but get very little idea of what the subject stands for and of its economy in the domain of mathematics. This could be avoided to a great extent, if the teacher himself had a clear idea of what algebra means. The same is true of geometry, where we are still hide-bound in the matter presented, much of which is in our books simply because it has always been there from the time of Euclid.

But merely to familiarize the student with the fundamental facts and operations of elementary mathematics constitutes only a part of the work of the teacher. If this were all, algebra and geometry might very well be dropped from the curriculum of all high school students, excepting from that of the few who are planning to go on with work either in pure mathematics or as engineering students. But we know that this is not all, and that the half has not been said. In fact, the student must learn

how mathematics enters into all the pursuits of man, how it enables us to understand better the laws of nature and how to express and study them. Moreover, he is to acquire from the study of mathematics a certain mental power that will enable him to see more clearly the relation between cause and effect, that will train him to logical reasoning, in short that will enable him to think for himself in an independent manner. Unless he has learned this, unless the study of mathematics has taught him in some way to stand on his own feet instead of constantly leaning on others for support, it will have failed of its purpose. Simply learning fundamental mathematical facts and operations will no more effect this than will a knowledge of the multiplication table alone make of one an arithmetician. A knowledge of these fundamental things is, to be sure, a necessity, a necessary evil, if you wish. We must, for instance, know how to handle algebraic fractions, how to factor, and how to perform other elementary operations accurately and quickly, because they enter so constantly into all future work. This, the mechanical side of the subject, is thus a means to an end and not the end itself, and as such it should be reduced to a minimum. But if a boy is to be interested in the work and is to succeed in mastering it, then he must know, at least to a certain extent, why he is studying the subject and he will ask to see some practical use for it. In this he may be satisfied in mathematics as in no other subject. In the study of geometry, for instance, instead of having so much of his time wasted on a mass of useless theorems, he should be taught the practical application in everyday life of the fundamental theorems. At least half of the theorems in the ordinary geometries might very well be cut out and replaced by more useful matter. The claim that we need these to prove other theorems is not valid, since all the theorems of geometry can be proved by means of half a dozen simple principles. A thorough knowledge of these principles and their applications will prove of greater use than a mass of undigested theorems. But if the teacher is to teach geometry in such a rational and practical manner, then he needs at least a knowledge of analytic geometry, and a further knowledge of the fundamentals of descriptive and projective geometry will do much to open his eyes to the possibilities of this branch of investigation.

With such special training in mathematics, as outlined, the teacher would be able to stand on his own feet, instead of lean-

ing on the book, his teaching would become more spontaneous and be free from that labored procedure which we so often find. He would have the power to inspire his students, and to awaken in them a true interest in the subject, because he himself would realize clearly how important an element it is in practical life. Moreover, we might then begin to expect a better class of text-books, better because it would not be necessary to work out all the minutiae of a subject, as now seems to be the case.

We have seen what would constitute a reasonable minimum of special training for the teacher of mathematics. Let us now investigate briefly the actual preparation of the teacher of mathematics in this country. It is a difficult matter to obtain accurate statistics on this subject, but the following are conclusions drawn from observations and inquiries extending over a number of years.

In the high schools of the larger cities, like Chicago, most of those teachers of mathematics who have charge of the department, have had considerable mathematical training, in some cases having their doctor's degree in mathematics. But even here, we find inadequately prepared teachers in the subordinate positions. Some years ago the head of the department of mathematics in one of the larger high schools of Illinois told me that one of his greatest difficulties was to secure well prepared subordinates. Too frequently the board required some teacher who might be well prepared in some other branches, as for instance in languages, to take a class in algebra, in order that she might be teaching sufficient hours to earn her salary. The next group of schools consists of those which may not be called larger high schools, but which are still large enough to have the department of mathematics differentiated with a teacher at the head of it. In these schools there is a tendency to secure such teachers as have had at least some slight special training in mathematics. I have recently taken occasion to make inquiries of a gentleman connected with one of our largest teachers' agencies, and he told me that a large number of their candidates for such positions have had the so-called freshman algebra and trigonometry, but have not had any analytic geometry. The last group of schools, and numerically a very large one, consists of the smaller high schools with from two to four or five teachers. In these mathematics is usually taught by teachers

who have had no special training in the subject. In perhaps the larger number of cases, those who teach mathematics in such schools have no more mathematical training than they themselves acquired in the high school. Summing up, we are probably pretty near the truth, if we say that in those high schools in which the department of mathematics is differentiated from the other departments, the majority of teachers have not had mathematics to include analytic geometry, while in the rest of the schools they fall far short of even this.

Need we be surprised, therefore, that under these conditions the product of mathematical teaching in this country has not been of the best? In fact, it speaks well for the good sense and energy of our teachers that it is as good as it has been, and we may confidently predict a bright future in this department of educational work, when we have once secured teachers who are thoroughly well trained in mathematics. How to effect this, is a question which should engage the earnest attention of all who are interested in mathematical teaching. The question is a difficult one, and in this country its solution will necessarily be slow. In countries like France and Germany, the problem has been fairly well solved, but there it was comparatively easy on account of the centralized school system. Under this, demands could be made and enforced, which could be neither made nor enforced under our system. Then the advantages of an assured competence and pensions have made the profession of teaching much more attractive for bright young men in the older countries than it is with us. In this country, where every board is a law unto itself, obvious difficulties arise. We are all familiar with these, and they need not be mentioned. And yet, I believe that much can be done to bring about an improvement.

A good deal is already being done toward securing a better trained corps of mathematical teachers. Thus, a number of our leading universities and colleges have inaugurated special courses for those students who are planning to enter the mathematical field as teachers. Foremost in this line are the Teachers' College of Columbia University and the School of Education of the University of Chicago. It will pay every teacher of mathematics to study carefully the bulletins in which the work of these schools is set forth. Other institutions are working along similar lines. Some are requiring a certain minimum amount of special

training in mathematics of those students who desire the recommendation of the department as a teacher of mathematics. All of our universities and colleges should eventually do this. There is no doubt that such a stand will have an excellent and beneficial effect. All our higher institutions of learning should offer courses that are specially adapted to the needs of those who are planning to enter the teaching profession in the mathematical field. This can be done without in any way detracting from the work of the department in pure and applied mathematics. At least one course on the history and the pedagogy of mathematics should be offered. I believe that our universities and colleges are not living up to the full measure of their possibilities, when they do not make some such provision in their curriculum. We frequently hear professors in the higher institutions criticise the preparation of the students who come to them from the high schools. No professor, who does not make some provision in his course for the needs of those who want to teach, has a moral right to criticise the work of the students sent him by the high schools. Let our higher institutions of learning grasp their opportunity to its fullest extent, and the cause for future criticism will disappear.

In some such manner as outlined, steps can be taken which will produce a future generation of better prepared teachers of mathematics. Something can also be done to help the present generation of teachers.

Among those agencies which can be most helpful to the present day teacher of mathematics, are again, in the first place, our higher institutions of learning. Their opportunity is found in the summer schools which are now being conducted by so many of them. Unfortunately, most of them have not begun to realize the great opportunity offered to them. In looking over the circulars of a large number of summer schools, I find that very few of them offer any courses that will be of special helpfulness to the teacher of mathematics who may be attending the school. Most of them confine themselves to the regulation courses of more or less advanced work, thus making the school a school only for backward students or for those who wish to take some work toward a degree. It is fairly obvious that the ordinary course, in which the regular work of a term or semester is crowded into six or eight weeks, sometimes by devoting double time to it, is of little use to the average teacher

in his class room work. The summer school should certainly offer a course on the history and pedagogy of mathematics. It should moreover give such courses on elementary algebra and geometry and give them in such a way, as will be of actual help to the teacher when he returns to his work in school. Very little is done along this line. The emphasis of the practical applied side of every subject would be of great help to the teacher, and this should be particularly emphasized in summer school courses. It will not in any way hurt the student who is not a teacher or who is not planning to become a teacher. In other words, the outlook of the teacher can be broadened. By teaching trigonometry, we can open up for him new vistas in elementary algebra and geometry.

While the higher institutions of learning thus have a field and an opportunity for aiding the present generation of teachers, there are other agencies which can do much in this line. Foremost among these is the superintendent of schools. He is the natural adviser of his teachers and should see to it that they keep a live interest in their subject and do not fall into ruts and grow stagnant. A young superintendent of my acquaintance found that a good many of his teachers were in this condition and were actually in danger of becoming wholly incompetent. Instead of dismissing these teachers, he argued that their experience was worth much to the schools and that they would become more valuable than inexperienced teachers if they would undergo a process of rejuvenation. So he advised them to attend summer school, in some cases going so far as to make this a condition of their retention. Now every summer a goodly number of them go off to study, and in this the superintendent himself sets a good example. The result is that the work in the schools under his jurisdiction is constantly growing better.

But there is another field in which the superintendent can be active. In one city of Illinois, the teachers have formed a local society at the suggestion of the superintendent. Each member contributes a sufficient amount of money each year to enable them to have talks by good men from outside. These talks are on various subjects, but such as would interest the majority of the teachers. Thus, a talk on the place of geometry in the grades, showing how we can make use of squared paper, paper folding and other means to cultivate in the children geometrical notions and intuition, is of interest not only to the teacher in

the grades but also to the high school teacher. New ideas are thus brought in and a general awakening results.

My observations have led me to the conclusion that, as a rule, our teachers are anxious to improve, but that frequently they do not realize that they need to improve very much. Let them once realize that they need improvement and that a greater knowledge of mathematics will enrich their teaching, and they are willing to take steps to acquire this knowledge. Our problem is to reach them and bring them to a sense of realization of the necessity of such additional work. It is especially the young man and woman who are teaching mathematics in the smaller high schools, whom we should try to reach. Here all can lend a hand. Each of us has opportunities of meeting such teachers and can exert an influence for good. We can call their attention to meetings of this kind and to the advantages of keeping in touch with what is being done to advance mathematical teaching. In this assembly, we find the pick of the teachers of mathematics in this vicinity, men and women who know their subject, who have learned to think for themselves, and who can therefore enter into a fruitful discussion of the different phases of mathematical work. But there are only a small number of high schools, even of Illinois, here represented. While we are here enjoying the privilege of getting together, of interchanging ideas, and of thus acquiring new vigor and new ideas to take back to our work, the teachers in the small schools are groping in darkness, and are wrestling almost hopelessly with the very problems which we are helping each other to solve. We should, therefore, be impressed with a personal responsibility in the matter and should feel it our duty to do some kind of missionary work along this line, extending a helpful hand of fellowship wherever and whenever we can. I think that much can be done by such personal work on the part of the members of an association of this kind.

SECONDARY MATHEMATICS.

BY R. L. SHORT.

Mathematical Editor for D. C. Heath & Co.

Until some five years ago little had been done by the mathematical world toward either the pedagogy of mathematics or the correlation of related subjects. For the most part teachers were content not to progress. Authors had strengthened up their text-books to some extent, some demand had arisen for more concrete problems and a few teachers believed in the graph. It took Prof. Perry to set in motion a scheme which has largely changed our ideas of mathematical teaching. The promoters of his ideas swung too far and urged upon teachers a plan which would only succeed under ideal conditions. These men practically urged the abandonment of operative algebra and the substitution of the workshop and measuring rod. This agitation accomplished much that was good. It aroused thinking, progressive teachers to action, aroused antagonism, brought many new and sane ideas into teaching of mathematics, brought together the science and mathematics teacher, took the various subjects out of their several compartments and correlated them.

And now the result? These prime movers, with one or two exceptions, have settled down to a conservative basis, especially so far as algebra is concerned. The algebra of to-day means more to the student. Transposition is no longer a name. It is a process dependent on familiar axioms. The fundamental operations are the operations of arithmetic, generalized. The equation is the central thought for which the operations prepare the way. The x, y, z, of a decade ago have lost their exclusiveness and the physics teacher complains less than formerly that his student cannot solve for t. And as for the graph, the man who now stands up and shouts that the graph does not belong to algebra, shouts alone.

One danger still confronts the algebra teacher. The pupil is so interested in the practical side, the pictorial side, the geometric side if you please, that this often causes neglect of the operative side, i. e., formal algebra. The development of the thought process is necessary, the applications of algebra are necessary. Thought processes are carried on in various subjects throughout the high school course. Are they to weaken formal algebra? Practically all the mechanics of algebra must be taught during

the first year of the high school. A large amount of drill work is necessary during this first year. How much time shall we give up to other things?

Up to this time geometry has been little disturbed. For this there have been two reasons. The colleges, technical schools and the physics teachers have not complained that the student is deficient in geometry. Why? The student's mind is more often geometric than algebraic and the student needs no geometry in his work. A little mensuration, mostly learned in the grades, and a few additional theorems picked up in the secondary school amply serve his purpose. Some attempt is being made, and rightly, too, to reach geometry in a less formal way, to take away some of the interdependence of theorems and to leave the student more to his own resources in the development of theorems. This has two effects, partially good, partially bad: The beauty and continuity of the subject are impaired, the student knows fewer theorems, has more thought power and can use his theorems to greater advantage.

The tendency to favor algebra at the expense of geometry will probably have a marked effect on our courses in mathematics. Some go so far as to declare that the year and a half each now devoted to algebra and geometry will soon give way to a course in secondary mathematics, similar to those now offered in Germany and Italy. Such course would doubtless give us students better prepared for college, better prepared for business and with less dislike for mathematics. It should produce more mathematics teachers, of which there is now a dearth, and should consequently improve mathematical conditions in both college and secondary school.

FACTORING THE TYPE px^2+qx+r.

By Byron E. Toan.

Boulder, Montana.

Of all types this is the most important, not alone because of its frequent occurrence but also because it includes those common forms x^2+px+q, $x^2+2xy+y^2$ and x^2-y^2. In fact, it is the general form of the product of two binomials, each of which has a term of the first degree in x. Because it is the most important type met in factoring, its proper presentation to beginning classes

should be a matter of considerable care and thought on the part of the teacher.

Factoring is taking the back track from the product to the factors that produced the product. It is *unmultiplying* the product. In multiplying two binomials, $ax+b$ and $cx+d$, four terms, $acx^2+bcx+adx+bd$, are obtained which combine, by algebraic addition, into three terms, $acx^2+(bc+ad)x+bd$. Inspection of this general product and comparison with the type-form, show that q, the coefficient of x, is the sum of two terms, bc and ad, whose product, $abcd$, is also the product of the coefficient p and the third term r, for $p=ac$ and $r=bd$. Now this product, pr, is a known quantity and if it can be resolved into two factors (in this case bc and ad) whose sum is q, the trinomial is factorable; otherwise it is not factorable. These two factors bc and ad, are the coefficients of x which, in the multiplication were added to give q and, when found, enable one to change the trinomial back to the quadrinomial from which it came. This change is the first and all-important step on the back track that leads to the factors. The quadrinomial, when determined, is easily resolved into its binomial factors.

An example: Factor $12x^2-11x-5$.

$$12\times -5 = -60$$

Two factors of -60 whose sum is -11, are -15 and $+4$.

$$12x^2-11x-5=$$
$$12x^2-15x+4x-5=$$
$$3x(4x-5)+1(4x-5)=$$
$$(4x-5)\ (3x+1).$$

This is, step by step, the reverse of multiplication for:

$$(4x-5)\ (3x+1)=$$
$$3x(4x-5)+1(4x-5)=$$
$$12x^2-15x+4x-5=$$
$$12x^2-11x-5.$$

NOTE:—This method is given as Case VI, § 117 p. 85 of *Algebra for Secondary Schools*, by Webster Wells. Being the edition of 1906, it is not strange that the author presents this paper here.—MATHEMATICAL EDITOR.

OPTICS BY THE WAVE METHOD.*

BY A. L. CAVANAGH,

Los Angeles High School.

During the progress in the development of the teaching of physics, light seems to have been side-tracked and so has come down to us as a sort of heirloom of the long ago. Our geometry differs as much from the geometry of Euclid as this treatment of light differs from that of the Arabs.

The caloric theory has altogether been displaced by the kinetic theory and this substitution has passed into history. The "projected particle" theory is being displaced by the wave theory and this is history in the making. In fact, the only thing left to do to complete the transformation is to modernize the teaching of the elements of the subject. The belief that this change will soon be brought about is strengthened by the recent arrival of text-books which attempt to treat light as a wave phenomenon.

It would be interesting to know how many of the physics teachers have made a careful study of the two methods in light and of these how many have been converted to the new.

In the few minutes given to this subject, I want to tell you what my experience has led me to believe after teaching both methods during the past five years. I have been asked to talk on this subject and in dealing with it I give my views not with a feeling of antagonism toward any one but with a wholesome respect for those who think differently after careful comparison of the two methods.

I know well the difficulties that a student must meet in attempting to acquire—if he should succeed at all—a true understanding of light propagation by the study of rays which points the immature mind only too surely to the antiquated theory. In fact, is is only after a study of Optics with "rays" left out that I have come to feel that I understand what I am trying to teach them. An image of the wave ought always to be uppermost in the mental picture.

I believe the method in geometrical optics to be unsound. I make this statement emphatic in order that my position may be understood. Some of my reasons for giving up the study of Optics by rays are here enumerated:

*Read before the Pacific Coast Association of Chemistry and Physics Teacher Dec. 1909.

1. The method is unintentionally misleading. However good the intentions of the teacher may be, most students are left with a hazy understanding of the subject, at best, because the bald facts are not kept constantly before them. In the mental picture the ray is substituted for the wave, which condition is not true to nature as we know it.

2. The method is too apt to leave the mind with a seeeming explanation of reflection by the rebounding ball idea—projected particles. The rays are bent at the reflecting surface for the same reason that the path of a billiard ball is changed abruptly when the ball rebounds from the cushion and the angle of incidence is equal to the angle of reflection for a similar reason.

3. The meaning of refraction cannot be made clear nor is the reason for it evident. The use of trigonometric functions before they are understood is undesirable, because it obscures the main idea. This is true of any process that takes the mental effort away from the main facts.

4. The study of rays does not lend itself so advantageously to the cultivation of the imagination as does the study of waves.

Every teacher must needs paint mental pictures for the student showing how the disturbance from any source travels outward in all directions and the light and heat we receive from the sun is just the small part of the whole that the cross-section of the earth is of the area of a sphere ninety millions of miles in radius. In brief, our first endeavor is to aid the student to imagine these growing spheres which represent the propagation of energy not only from the sun but from any body radiating heat and light. From this point of view can be developed practically all of Optics. Even so much as the mention of the words ray, pencil, etc., here are ill-advised because of the tendency to obscure the true facts. Insist on the word radius instead of ray and translate everything in terms of these spheres and so keep this idea uppermost.

In terms of spherical surfaces, the law of inverse squares as used in photometry is easily understood. And I find reflection and refraction to be no more difficult of explanation. Let us at least try to give a true conception of reflection. The bending of rays and the production of these to intersect for the purpose of locating the virtual image have their short-comings. Why do the rays bend as they do? Why is the angle of in-

cidence equal to the angle of reflection? Is it because a tennis ball rebounds from a wall in some such manner? We expect the student to be satisfied with an answer that is no answer at all. Let us encourage intelligent questions and give as intelligent answers as we can.

We cannot build without axioms from which to begin. The axioms of the wave method are simple and reasonable. In fact they are accepted, in different words, in the older method. These axioms are: 1. Any object is seen at the center of curvature of the energy waves we receive from the body. 2. These waves are approximately spherical in a homogeneous medium. The second is implied in the first but is here stated for purpose of construction. The second must needs have limitations.

When a wave-front falls on a polished surface it is reflected as a wave-front. Its velocity may or may not be changed by being turned back. If the velocity isn't changed it is evident that the curvature is reversed but not changed in value. It will seem to be coming from a point back of the reflecting surface and this point is the image of the true source. In a plane mirror the object and its image are equidistant from the mirror. If the velocity were changed by reflection, the curvature of the reflected wave would be changed and the image would appear either nearer to or farther from the mirror than the object is. In this way nothing need be said about angles of incidence and of reflection.

The construction is equally simple for reflecting surfaces of any form so long as they are spherical because a drawing represents a cross-section only. I find that the students make these constructions with little assistance and many of them with no assistance at all. This, of course, after the construction for the plane mirror is understood.

Having no text-book this year, I assigned many of these constructions as "originals." I had them use a slanting arrow as object and follow waves from each tip of the arrow and so obtain the position, size and slant of the image. I found many of them could do this correctly though they had no reference books or old folders from which to get suggestions. The following figure will illustrate the method when the mirror is convex.

In this figure AB represents an object in front of a convex spherical mirror MN. The spherical wave fronts from A and B are reflected from the mirror as spherical wave fronts which to an observer anywhere in front of the mirror, appear to come from A′ and B′ respectively. Since these points behind the mirror are not true sources of light waves, A′B′ must be a virtual image of AB.

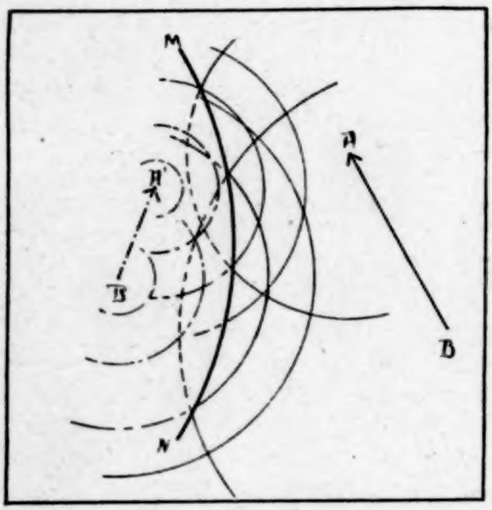

Fig. 1

The WHY of things is more evident with the method of waves. I have seen students locate images at certain places "because the rays crossed there." The meaning of the crossing of the rays was lost. If the waves are drawn, it will be seen that where the rays cross the wave-front contracts to a small area or focuses. If a screen is held at this place, a point on that screen receives all the light starting from a particular point in the source and reflected by the mirror or refracted by the lens. This is the condition necessary for the formation of a real image.

Again, a coin in a dish of water appears nearer to the surface than it actually is "because the rays are bent from the normal and, when produced, the virtual image is located at a point higher in the water than the object is." This doesn't explain the bending of the rays and so the explanation is not satisfactory. As a matter of fact, the reason is quite easy to

see in terms of waves. The wave travels faster in air than in water so the wave-front upon emerging from the water becomes more convex which makes it appear to come from a point higher in the water.

There is no reason why the wave-method should be avoided because of the difficulties in its presentation. This year our text-book consisted of laboratory notes which I purposely made rather full. However, the figures were not drawn for them and only suggestions were given to outline and facilitate the work. I had them draw the figures before they knew what they were to find and in this way they arrived at the conclusions which were later verified by experiment. The work on the whole was a pleasant surprise to me. Refraction offered no more difficulties than did reflection.

Refraction is not a bending of the rays any more than reflection is. Refraction is the changing of the shape of the waves as they pass from one medium into another when the velocity of light is different in these two media and the change in shape is due to the change in velocity as is easily seen. If the velocity of light waves is different in two adjacent media, the wave-front in passing from one into the other must, in general, become more or less convex and this will cause the source to appear either nearer to or farther from the observer, depending upon existing conditions.

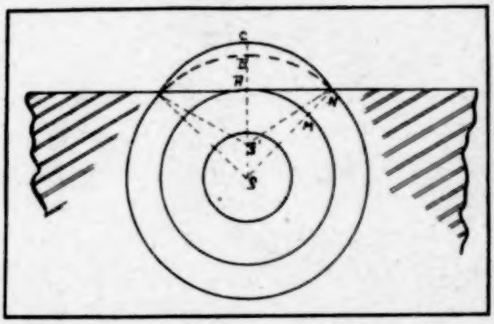

Fig. 2

The measurement of the refractive index becomes a simple and easily understood problem from this point of view. The trigonometric functions are unnecessary, which is certainly a point in its favor. My construction for this is essentially that

given in the text-books though I believe I have simplified the explanation somewhat:

In the figure, S is a source of light in some medium other than air. As seen by an observer, it appears to be at S′. The waves emanating from S are spherical until the surface is reached where the curvature becomes more convex as shown by the approach of the virtual image to the interface. The virtual image S′ is the center of curvature of the waves in air.

It is evident that while the wave-front has been growing so as to increase its radius by a distance MN in the denser medium, in air the radius SA has been increased by an amount AC. That is, the wave travels a distance AC in air in the same time that it would require to go a distance MN in the other medium. Whence

$$\frac{\text{the velocity of light in air}}{\text{the velocity of light in the other medium}} = \frac{AC}{MN} = \frac{AC}{AB} = \text{R. I.}$$

By this method the Refractive Index comes to be intelligible when expressed as a ratio of velocities rather than a ratio of the sines of angles. It becomes a simple task to measure this constant for any transparent substance. For liquids I use a rectangular battery jar and the results are decidedly satisfactory.

The effect of lenses on the shape of the wave-front is readily understood. It is evident that a wave-front in passing through a lens is retarded by the glass and that part must be retarded most which goes through the most glass. So if the lens be convex, the wave-front is made less convex; if the lens be concave, the wave-front becomes more convex on passing through the lens. The same simple reasoning applies to the prism.

One of the most difficult things for a student to understand is the arrangement of lenses in the compound microscope and telescope. Here the drawing in of the waves at different places makes the explanation sufficiently plain. Compare the two following figures to illustrate this point. One is drawn entirely with rays and the other entirely of waves.

In the figure, AB, the object to be viewed through the compound microscope, is placed just outside the principal focus of the objective. The waves from A and B, after passing through the lens MN are concave and focus as shown at A′ and B′, respectively, so as to form the real image A′B′ within the principal focus of the eyepiece RS. The image is inverted and enlarged.

The real image being a source of light sends out spherical waves which emanating from points within the principal focus of the eyepiece cannot become concave by passing through RS but are made less convex. An observer receiving these waves sees an image at the center of curvature of these waves, at A″B″.

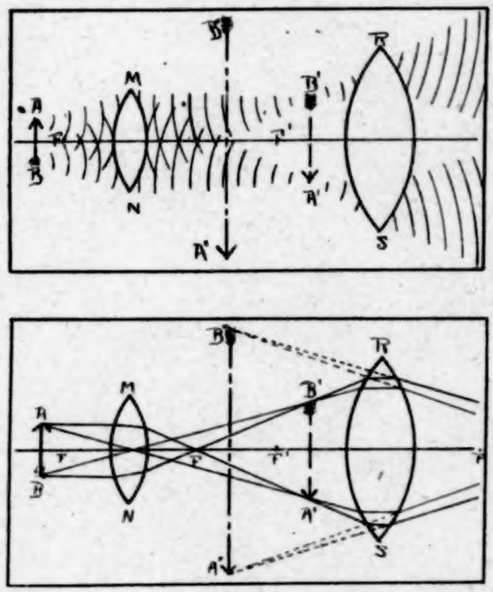

Fig. 3

The constructions seem to interest students, especially those in mechanical drawing classes. The interest is perhaps due to the fact that the compasses are used a great deal more than in the other constructions. As evidence of this interest I have a collection of plates which have been drawn and contributed by these students and this collection is added to each term. It is my intention to have these enlarged for wall charts.

It would be strange if the method offered no difficulties whatever. I believe it offers fewer than the ray method and one of these is total reflection. But the old method also seems to be misleading in this particular because it gives the impression that vision is perfect untile sine r=1. Let me quote from a letter received from Prof. Sanford touching on this point. He says: "I am skeptical about there being any part of the emergent wave front exactly perpendicular to the surface of the separation

of the two media, though it is possible that some scattering light emerges parallel to the surface. I have tried to measure the angle of the refracted ray when it disappears, and I have never been able to get any appreciable light at a greater angle than 89° with the perpendicular to the surface and no object can be seen within several degrees of the surface."

I believe in the wave method in Optics because it leads to and is based on true conceptions; because it is more explanatory in its teachings; because it is more easily understood and retained by the student; because it is more interesting and because it permits of original work on the part of the student.

The method of rays has served its purpose well. But as the undulatory theory has replaced the belief in projected particles, so I believe the constructions based upon that antiquated theory ought to give way to what is at least nearer to nature.

To those who advocate a proper mixture of the two methods I would say that there is danger of confounding the student because of the brief time allowed for the subject. But were I to undertake to give such a course I certainly should begin with the wave method.

There may be those who hold the method in disfavor because it does not lend itself well to quantitative demonstrations. While admitting that definite relations may be obscured by errors in construction by the wave method, I would answer that in the necessarily brief time allowed this subject in the year's work, I believe the approximations to be obtained by this method to be sufficient.

AN APPARATUS TO ILLUSTRATE SOME DEFECTS OF THE EYE AND THEIR REMEDY.

By Carl J. Ulrich,

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In studying myopia (short-sightedness) and hypermetropia (far-sightedness), I have found a piece of home-made apparatus very helpful. It will be remembered that in myopia the trouble is usually caused by the fact that the eyeball is too long from front to back, thus causing the retina to be some distance behind the focal point of the lens. Hence the image from distant objects, the rays of light from which come to the eye parallel or nearly so, are brought to a focus some distance in front of the retina. Near objects, with diverging rays, are seen plainly, because, with an equal amount of refraction as compared with parallel rays, the diverging rays are focused on the retina. The means of correction of myopic eyes are concave lenses, which cause the parallel rays of distant objects to diverge, thus bringing the focus on the retina.

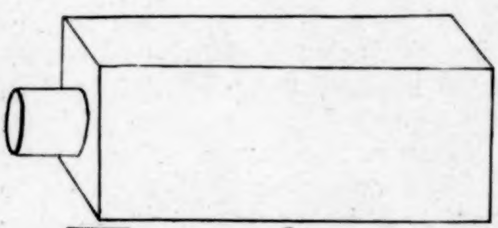

Fig. 1

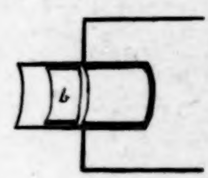

Fig. 3

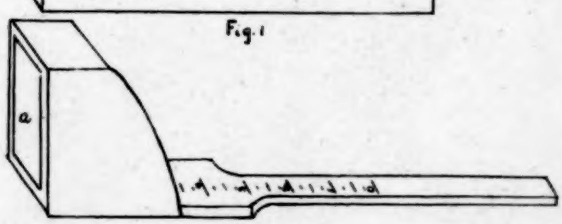

Fig. 2

In far-sightedness, the eyeball is usually too short, so that the focus of the diverging rays of near objects falls behind the retina, while the parallel rays of far objects can usually be focused on the retina. The means of correction for far-sightedness are convex glasses, which bring the focus of diverging rays forward, so that the image is formed on the retina.

The apparatus used consists of a rectangular box 9 by 3½ by 3½ inches, open at one end, while fastened to the middle of the opposite end is hollow cylinder 3 inches long and 1½ inches in diameter (see Fig. 1). The bore of this cylinder is reduced slightly in the half nearer the box, so that lenses slipped into it rest against the flange thus formed. A ring ½ inch wide and large enough to just slip into the cylinder is used to keep the lenses from falling out (Fig. 3, b). Fitting into the box is a sliding portion which is a section of a box of such size that it will easily slide into the rectangular box that serves as the camera. The one end of this movable part is provided with a square of ground glass (Fig. 2, a), while the other end is open. A handle, graduated, is attached to it. (See Fig. 2.)

The lenses used belong to a set of demonstration lenses, 6 in number, sold by the Bausch & Lomb Optical Co. for about $1.00. In the set are 1 double convex, 2 plano-convex, 1 double concave, 1 plano-concave, and 1 meniscus-concave lenses. The double convex and plano-convex lenses have practically the same focal distance, while the concavity of the plano-concave lens is about equal to the convexity of the plano-convex lens.

To illustrate myopia and its correction, place into the lens holder 1 plano-convex lens, then adjust ground glass so that the image of some distant object, say a building, appears clear and distinct. Read the focal distance as shown on the graduated handle. Call this the position of the retina in the near-sighted eye. Now add another plano-convex lens. This will shift the focus forward. Move the ground glass forward till the image becomes clear. Read the focal distance, and call this the point where the focus falls in the near-sighted eye. For instance, if the retina is 6 inches from the lens, and the focal distance as we now have it is 3 inches, then the image in this eye is 3 inches in front of the retina. Now to correct the defect, add a plano-concave lens, and the focus will be at 6 inches, the position of the retina.

In a similar way, the correction for long-sightedness may be shown. Put in the 2 plano-convex lenses and call the focal point the position of the retina. Then by the addition of a plano-concave lens the focus is moved some distance back, which is the condition in a far-sighted eye. This is corrected by the addition of the double convex lens, which brings the focus forward again.

A REMARKABLE PHENOMENON.

By C. E. Peet,

Lewis Institute, Chicago.

While on a canoeing trip in the forested wilderness of Ontario this summer I had the pleasure of witnessing the following remarkable phenomenon: At about four o'clock in the afternoon of a day in August, while we were paddling along the west shore of one of the lakes, which are so numerous in Ontario, a thunderstorm passed over us, and in its rear we beheld the not uncommon phenomenon of two rainbows spanning the sky. Imagine our surprise, however, when a third rainbow suddenly flashed into sight inside of the other two and an instant later a fourth appeared inside of the third. The three inner bows were so brilliant and the contrast between their light and that of the sky was so great we had hopes that they would show in a photograph. Unfortunately my films had been injured by dampness and I did not succeed in getting a picture. My astonishment was so great and my attention so taken up with the beauty of the scene that I neglected to take account of the arrangement of the colors in each bow. The inner bows were conspicuously nearer together than the outer. The space between the bows was progressively greater from the inner one to the outer. I was left with the impression that the curvature of the inner bows was conspicuously flatter than that of the others, but of this I am not perfectly certain. Hastings & Beach make the statement in their "General Physics," that the third and fourth bows have never been seen.

In the "Weather Review" for July, 1905, there is the following account of a quadruple rainbow:

"Ciel et Terre publishes the following account of a quadruple rainbow observed at Mons, Belgium, August 31, 1904, by M. A. Bracke:

"A shower had just ceased (17 h. 55 m. to 18 h. 30 m.) when a superb rainbow appeared—a complete semicircle, very broad, with magnificent colors, presenting distinctly the seven hues of the spectrum. A little above, less distinct and less broad, was the segment of an arc showing the red, the yellow, and the green. Below, and very near the large arc, was a third, having a breadth about equal to a quarter of the last, and showing but two colors—mauve-red and green. Finally, beneath this same arc was distinctly visible, but at intervals only, a fourth, colored like the one above it. The phenomenon lasted a quarter of an hour."

FIELD WORK IN PHYSICAL GEOGRAPHY.*

By Jennie T. Martin.

Central High School, Buffalo, N. Y.

There is no need here to consider the importance of field work in Physical Geography, as its value is recognized by everyone who teaches the subject. I shall try instead to outline for discussion by the section, some of the problems connected with the work while presenting plans and methods carried out in my own classes. These plans have gradually developed during a series of years, and are still changing through the constant effort to overcome obstacles and obtain better results.

There are two important aims to keep before one in this work, first, to make it so enjoyable that a life-long impulse toward the study of nature in her great out-of-doors laboratory shall be gained, and second, to make it so effective that it shall minister directly to the mental development of the pupil.

There are many difficulties to contend with, especially in city schools, the large number of pupils to be looked after, the long distances often necessary to cover in order to reach points of interest, the expense involved, the weather, and greatest of all the inability of pupils to use their eyes or to understand the meaning of what they see. One of the greatest benefits coming from field work I find to be a dawning consciousness on the part of the pupil, that he may discover facts for himself, and that facts so learned are as authoritative as those he finds in books. It is very difficult at first to get pupils to rely on themselves and to depend on their own conclusions. They try hard to get me to answer their questions for them. With their directions in hand as we pass from point to point they repeatedly ask "What are we to see here?" The reiterated hint to "Look at your notes" and the patient use of the Socratic method, with now and then a little direct help, finally however make them somewhat self-reliant and a great victory is won.

In planning excursions I have worked mainly to make them effective, believing that this in itself will bring pleasure.

For the early excursions detailed directions and definite questions seem necessary. But in using them, I have found on the part of pupils a tendency to be satisfied when they have made the required observations, so that they do not see much for

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*Read at the New York Teachers' Science Association, December, 1906.

themselves. For this reason I usually call for two or three original observations and mark the pupil if these are lacking. Toward the end of the year I give the pupils more freedom and find that better results are obtained by outlining general points for observation and report. Sketches and drawings test the pupil's knowledge better than words and should be called for constantly. The camera is also a great aid.

Where the Geological Survey topographic maps are available, they should be in the hands of each pupil, and routes should be traced on them and relief features identified. It is no small gain for pupils to learn their practical value. We have five pocket compasses for field use and two good aneroids. Pupils need pocket clinometers, which can be made by fastening a small plumb line to a protractor. It is good training for them to be required to be as accurate and definite as possible and to learn the use of field instruments. In the coming spring I expect to teach pupils the use of the hand level. We have copper plates of a few interesting features seen on our excursions and we furnish prints to our pupils for a small sum.

The preparation for an excursion takes considerable time and we plan in the near future to have printed directions, perhaps in the form of a field pad or note book. We shall thus save the time of dictating directions and obtain notes in better form.

To properly take care of the large number of pupils who go on our excursions is impossible for one person, and for several years I have chosen as field assistants three or four of the best workers in previous classes. It is then possible to form groups of about twenty pupils each and to assign an assistant to each group, who is able to aid the pupils and to keep them at their work. To call the groups together for general directions, or when I wish to explain something, I use an umpire's whistle. To render it easier for pupils to hear explanations, I plan to have a small megaphone.

There has been a noticeable gain in the quality of work done since I have required field notes to be submitted to me or to the assistants for approval in the field. Careless pupils have thus been prevented from making the excursion a mere pleasure trip with the chance of writing an acceptable report by borrowing the notes of some faithful pupil. It also has made it possible for those who have done poor work to do it over before leaving the place.

On the day following the excursion, the pupils bring their notes to the laboratory and write their report, using two laboratory periods for the purpose. The facts are then fresh in mind and this plan insures independence, and enables the teacher to help the pupil who fails to understand some feature of the field work.

The order in which excursions are taken seems to be important. My present plan is, in the Fall to have facts learned at first hand in the field and to follow excursions with book study of the same features, while the Spring trips have been planned especially to test and strengthen the observing powers of the pupil and to serve as a practical review of facts already learned.

For the first excursion we visit a quarry. This gives us a very limited area with few and simple problems. The pupils study and describe the rock structure, stratification, joints, synclines, anticlines, and weathering and bring back for laboratory study specimens of limestone, hornstone, shale and residual soil.

The same features are studied in other places in the next two excursions. This gives an opportunity for comparison and it also helps to correct false impressions gained on the first excursion. For example, I find a tendency for pupils to think that a joint is a peculiar phenomenon found in one part of one quarry, and the idea of a system of joints is not easy for them to grasp. Again, this plan enables them to realize the fact that the soil is everywhere underlain by rock, an entirely new idea to most of them. In these excursions in addition to the comparative study of the rock structure, one or two new features are added each time.

The most comprehensive excursion is saved for the last, in order that the pupils may get the most possible from it and that there shall be no anticlimax.

To the teacher who has not yet planned a set of field excursions, some hints as to how to set about it may be of interest. Use all available helps in getting acquainted with your region, the United States Geological Survey maps, state geological survey reports, and bulletins. Talk with those who know the region, if you can find such persons. Outline the features that are probably found in the region. Visit promising places that are not too difficult of access by students. Take notes on what you find, and make these notes the basis of directions to students.

Most regions in New York State furnish illustration of all the causes and results of weathering and erosion, of the various phenomena connected with young and mature drainage, of shore features as exhibited in lakes or ponds and of glaciation. I find it possible to have my pupils study in the field every topic under the land and land waters except mountains and volcanic action. The places most worthy of examination and giving the richest results will probably be stream valleys, quarries and other places where the rock is exposed, roadsides and excavations of earth, and the shores of ponds or lakes.

THE PRESENTATION OF THE IDEAS OF POSITIVE AND NEGATIVE NUMBER TO BEGINNERS.

ROB'T M. MCDILL.

New Castle, Indiana.

If we can show the need for negative number and even a few of its applications, the subject will be much simplified. The teacher will be compelled to consider four questions: What are the facts with regards to negative number? What material does the pupil have in mind, or easily in reach, which will enable him to grasp negative number? What particular questions or devices will help to bring out the laws which govern its use? Also wherein do mathematicians disagree with regard to negative number?

The class may be told that in arithmetic they have dealt with number from zero up to an indefinitely large number, while in algebra he must study the system from an indefinitely large (or as he will later call it small) negative quantity up through zero to an indefinitely large number. At the same time the teacher may have a line on the blackboard; this line may be marked off into units numbered both ways from zero, those below or to the left being marked minus. In this way the pupil is shown that zero may represent a starting place and not necessarily absolutely nothing.

The thermometer is always at hand in the school room and it furnishes probably the best illustration which the teacher can use for the first lesson. The pupil will be reminded that in the weather reports 10° means 10° above zero, while —10° means 10° below zero. A large percent of the class will be familiar with

this use of the minus sign. And even those not familar with the weather reports very quickly take to this method of representing temperatures above zero by plus and temperatures below by over the custom adopted by the weather bureau of representing temperatures above zero by plus and temperatures below by minus. And yet in this simple and natural illustration we have + and — representing not operations but qualities. While the pupils are looking at the thermometer and thinking of the weather reports, ask some one to subtract 8 from 6. He will look at you in amazement and say that it is not possible to subtract 8 from 6. Ask him if it is possible to become 8° colder than + 6° he will tell you that —2° would be 8° colder than + 6°. Tell him that this is what you mean by subtracting 8° from 6° and you have opened up a new field. After several such questions have been asked and the pupils have come to answer readily ask a pupil to add —8 and 12. If he is confused turn to your line on the blackboard and with him count up to 12 units from the point —8. You have gone a long way toward the answer when you have made the pupil see the meaning of the question. Several years ago I heard a university professor, now a well known university president, say in a talk on imaginary numbers: "Why, they are misnamed! They are not imaginary at all. They are just as real as any numbers." And so they were to him. He knew how they are obtained and how to interpret and represent them. So if we can make real to the pupils these simple numerical problems we have gone a long way toward the development of the laws which govern their use.

The pupil may be told that in the future work distance to the right of the starting point is called + and he will anticipate that distance to the left is called —. Tell him that distance down is called minus and he will anticipate that distance up is called +. In this way, one of the fundamental conceptions of analytical geometry is awakened in his mind: My subject, as originally assigned me, is the presentation of positive and negative quantity. But in the development of the laws of signs quantity must represent number,—a multiplied by b can have absolutely no meaning to the beginner until he thinks of these letters representing numbers. It is one of the ideals of algebra to make ideas, conceptions, principles and proofs *general*. But it is also true that we get general ideas from particulars. The mind works from particulars to general conclusions, and then on to particular again.

Therefore in the development of the laws of signs, with pupils who are really beginners, it will be necessary to use numbers for illustrations. This is especially true of addition and multiplcation.

There are four cases in addition, the addition of two positive numbers, the addition of a positive and a negative number, the addition of a negative and a positive number and the addition of two negative numbers. For practical purposes the first two of these cases may be ignored. The first because it has been treated in arithmetic and the second because from the nature of addition it is the same as the third.

The addition of a negative and a positive number may be illustrated by the blackboard line already described. Find the point represented by the negative number and count up as many units as there are in the positive number. Write each problem together with its answer on the blackboard. As soon as the pupils are able to add readily having before them the line marked off into units, call attention to the fact that each result is the same as would be obtained by subtracting the less absolute value from the greater and prefixing the sign of the greater absolute value. The problem is thus reduced to one of arithmetical subtraction. If pupils have trouble in remembering which sign belongs to the result, the teacher may suggest that they let majorities rule. The fourth case, the addition of two negative numbers may be well illustrated by the money illustration or by the one of gains and losses. The addition of —5 and —10 may be compared to the combining of a debt of $5 and a debt of $10. The pupil will readily say a debt of $15. And the fact that it is a debt may be indicated by prefixing the minus sign. Write many illustrations on the board, then call attention to the fact that in each case the result may be obtained by adding the absolute values of the numbers and prefixing the negative sign, to the result.

Then have the pupils commit the rules which may be worded as follows. To add a positive and a negative number, subtract the less absolute value from the greater and prefix the sign of the greater. Also, to add two negative numbers, add their absolute values and prefix the negative sign.

After the rules have been learned, let them be freely used. It is not necessary to have in mind an illustration every time two quantities are added. Although it is desirable to be able to give one if called upon. It is a mistake to think that the mind must

continually keep in conscious view every principle used. Each one of us uses freely the proposition, that "in the same or equal circles the greater of the two chords subtends the greater minor arc." We use this proposition even when its proof is not before the conscious mind. I do not mean that we could not prove the proposition if called upon, but I do mean that we frequently use the conclusion without thinking through the proof. So in adding —7 and 10 we do not always have in mind an illustration but we go ahead and use the established rule. What shall be done with the pupil who makes a blunder? Give him an illustration to bring out the blunder or have him review the rule? Probably he will need both treatments.

The subject of addition must not be left until it is thoroughly mastered, this is especially necessary because the problems in subtraction are so easily confused with those involving the addition of negative number. Then subtraction may be taken up. This will divide itself into two cases: The subtraction of a positive number and the subtraction of a negative number. The subtraction of a positive number may be illustrated as follows; on the line represented by the number system find the point represented by the minuend and then count down as many units as there are units in the subtrahend. Each problem may be written on the board and after several have been written show by parallel problems that subtracting a positive number is the same as adding a numerically equal negative number.

The subtraction of a negative number has often been compared to the taking away of a debt. The taking away of a debt of $5 has the same effect on a man's financial standing as the giving of $5 to him. And combining the two cases subtracting any number is the same as adding a numerically equal number with the sign changed.

Multiplication will divide itself into four cases. Expressed in abbreviated form by a + by a +; a — by a +; a + by a —; and a — by a —. The first need not be considered. The second may be illustrated by such problems as, What is five times a debt of $6? That is —$6 multiplied by 5? The pupil will readily say a debt of $30. And how can the result be written to show that the result is a debt? On one morning the thermometer stood at 3° below zero and the next morning it was four times as much below zero as the first. How cold was it the second morning? What is —3° times 4? After a series of different numbers are substi-

tuted in the above questions the pupil is ready to conclude that a negative number multiplied by a positive number is negative. It is not easy to get good illustrations of a positive number multiplied by a negative number. There are two things that may be done. We may say that so far as abstract numbers are concerned, it makes no difference in arithmetic, if the multiplier and multiplicand are interchanged. If the same principle holds in algebra we may say that a positive number multiplied by a negative number is a negative number. This argument is given in some form by a number of our authors. There is another way by which we may approach this case. Suppose we take the series of problems $\begin{array}{ccccccc} 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 2 & 1 & 0 & -1 & -2 & -3 \\ \hline 6 & 4 & 2 & 0 & & & \end{array}$ etc. Now suppose that in one line representing positive and negative number, we represent in order the first four products and ask the pupils what they would expect the fifth product to be, the sixth, etc. Your pupils will give you the correct results, and if they have come to look on 0, —1,—2, etc., as a continuation of real number down from a starting point, they will feel that a positive number multiplied by a negative number is negative. And this agrees with the result obtained by the first method. Considering the question from the standpoint of the above or some similar series may seem novel, but is it not similar to the method of finding the value of (1+)x when x is infinite? If we make x absolutely infinite we can do nothing with our function, but we attack the problem by finding the value which the function approaches as x approaches infinity and we call that the value of the function when x is infinite. I speak at length on this case as it is the most difficult with which we have to deal in any of the laws of signs. And I am just as willing as anyone to acknowledge that the above treatment is not satisfactory from the standpoint of technical reasoning, but I am not willing to give up the above methods until some one will suggest one that is better.

The fourth case may be illustrated by problems in the form 9—2 multiplied by —4. —4 times 9=36, —4 times —2 is numerically 8, put in a question mark for the sign of the 8, then make the expression equal to —4 times 7, that is, —28. Then let the pupils tell you what signe must replace the question mark in order to make the equation true. In this wya it is possible to force the pupil to tell you that a negative number multiplied by a nega-

tive number is a positive number. If he is a thinking pupil he will be surprised but so were you the first time you came to this result.

Some pupils may be helped by a series of problems in which the multiplicand is a constant negative number and the multipliers decrease by ones from say 4 to —4. The teacher who wishes to examine in detail the illustrations given by different authors for multiplication will not overlook those given by Aley & Rothrock, also by Beman & Smith. They are too long to quote here.

The signs in the four cases of division are readily determined by the pupil if he uses the same test for correct division that he he has used in arithmetic, namely, that the product of the divisor and quotent must equal the dividend.

The teacher will spend several recitations on the subject of this paper, and it will therefore be possible in class work to multiply illustrations. When the pupil is satisfied of the truth of any law of signs he should be compelled to commit the law. Take the law for subtraction for instance. If time is to be saved and the pupil to be in a condition to be helped by the teacher's suggestions, he must be able to give instantly the rule. In making the above suggestions I am not unconscious that they are arithmetical and inductive. Just as the proof given in arithmetic that the area of a rectangle is equal to the product of its base and altitude, is inductive, probably does not include fractions and is surely not such a proof as would be accepted in geometry and yet it does satisfy the mind as well as the latter proof. So the above proofs of the laws of signs are rather illustrations than proofs and are subject to many criticisms and yet, I for one, believe that they are proofs and that they will satisfy the pupils.

If we examine the subject of negative number as presented in the text-books of today we will find that the authors arrange themselves into two classes. In general the more elementary books attempt to prove these laws. Others represented by Chrystal lay down certain laws as fundamental in algebra, being careful that the laws be self-consistent and that they include as particular cases the laws of arithmetic. Then the operations are defined by the laws. That is the law of signs, for example, help to determine the meaning of multiplication as that subject is expanded beyond its meaning in arithmetic. These authors go

from the laws to the meaning of the operations and from the laws and operations to the applications. Let me quote from Chrystal: "It may be well to insist once more upon the exact position which they (the fundamental laws) hold in the science. To speak as is sometimes done of the proof of these laws in all their generality is an abuse of terms. They are simply laid down as the canons of the science. The best evidence that this is their real position is the fact that algebras are in use whose fundamental laws differ from those of ordinary algebra." In support of this he gives as an example the fact that in Quaternious ab=ba.

This confusion results from the following facts, the fundamental operations are very difficult of definition. And the definitions of discrete arithmetic are not satisfactory when we come to extending the number system. 4×5 is a matter of definition so far as the use of the word times is concerned, but if the number system is agreed upon and the word times is understood then problem becomes a matter of calculation. So if we start with certain notions as to how we shall apply the four fundamental operations in algebra, then the results are matters of calculation and demonstration not matters of definition.

There is always a chance that the order of the historical development of a subject is the pedagogical one. Diaphantes knew that "minus into minus gives plus" two thousand years before the process was reversed and "minus into minus gives plus" was made a part of the definition of multiplication. But what shall we say of the case cited by Chrystal Quaternious?

Again an historical incident helps us out. Sir William Hamilton did not start with the law that ab= —ba and develop the subject but he started with certain conceptions and interpretations and after long struggles found that as he was representing and interpreting quantities and operations ab ba as he had up to that time supposed.

Therefore I believe that with beginners we are right when we explain how we interpret negative number, and how we apply the fundamental operations and then make the laws of signs matters of demonstration. If the writers for advanced students can be more logical by reversing the process we surely have no objection.

PROBLEM DEPARTMENT.

IRA M. DELONG,

University of Colorado, Boulder, Colo.

Readers of the Magazine are invited to send solutions of the problems in this department and also to propose problems in which they are interested. Solutions and problems will be duly credited to the author. Address all communications to Ira M. DeLong, Boulder, Colo.

40. *Proposed by O. R. Sheldon, Chicago, Ill.*

A square garden, sides 12 rods, is planted with trees, no two of which are less than one rod apart, and no tree less than one-half rod from the fence. How many trees can be planted?

[NOTE.—Mr. Sheldon offered a prize of $3.00 for a correct solution which shows that more than 152 trees are possible; but no such solution has been received.—EDITOR.]

Solution by J. W. Ellison, Alcott, Colo.

Planted in squares one rod apart, no tree less than a half rod from the fence, there would be 12 rows of 12 trees each, 144 trees in all.

Planted in regular quincunx order (12 trees in the odd rows, 11 trees in the even rows), there would be 7 rows of 12 trees each and 6 rows of 11 trees each, 150 trees in all.

If now the quincunx order be used for the first 9 rows and the square order for the remaining 4 rows, there would be 9 (the last 4, and 5 out of the first 9) rows of 12 each and 4 (the even ones in the first 9) rows of 11 each, 152 trees in all.

43. *Proposed by H. C. Whitaker, Ph.D., Philadelphia, Pa.*

A heavy sphere of radius R is placed in a glass of water in the shape of a cone, radius R and height h. How much water runs over?

Note by H. E. Trefethen, Kent's Hill, Me.

Mr. I. L. Winckler's solution on page 139 of the February issue assumes, as he states, that the sphere sinks in the water, i. e., $r < \frac{R\sqrt{h^2+R^2}}{h}$. The limiting case, when D coincides with A, is interesting, the volume of water displaced being readily found from the relation $r : R = \sqrt{h^2+r^2} : h$.

But if $r > \frac{R\sqrt{h^2+R^2}}{h}$, the sphere is not then tangent to the side of the cone but rests upon the rim of the glass and its sides if produced would cut the sphere. In this case the formula given for finding EF fails and EF may be found from the equation $r^2 - R^2 = (r - EF)^2$ or $EF = r - \sqrt{r^2 - R^2}$, using only the minus sign before the radical since EF $< r$.

ALGEBRA.

48. *Proposed by I. L. Winckler, Cleveland, O.*

A and B run around a course, starting from the same point, in opposite directions. A reaches the starting point 4 minutes, and B 9 minutes, after they have met on the road. If they continue to run

at the same rates, in how many minutes will they meet at the starting point? (From Wells' Algebra.)

Solution by R. P. Harker, Parker, Ind.

Let x = distance traveled by A before they meet, y = the distance traveled by B, let t = time. Then $\frac{y}{4}$ = A's rate, $\frac{x}{9}$ = B's rate, and

$$t = x \div \frac{y}{4} = y \div \frac{x}{9}, \quad \text{whence } x = \frac{3}{2}y$$

Hence $x + y = \frac{5}{2}y$ = whole distance. Since A travels y in 4 minutes, he travels $\frac{5}{2}y$ in 10 minutes. Similarly, B travels the whole distance in 15 minutes. Therefore A and B meet at the starting point again in 30 minutes, the L. C. M. of 10 and 15.

GEOMETRY.

49. *Proposed by Byron E. Toan, Boulder, Mont.*

In a circle, radius R, given an arc of 45 degrees. To find the radius of a circle passing through the extremities of the given arc and having the area common to the two circles equal to 2/5 of the area of the required circle.

I. *Solution by H. E. Trefethen, Kent's Hill, Me.*

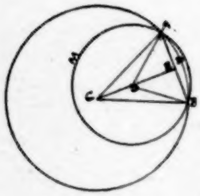
Let the center of the required circle be exerior to the given circle. Let r be the radius of the circle sought and x the angle between its radii drawn to the extremities of the common chord. Put $a = 45°$. The area common to the two circles consists of two segments separated by their common chord. The area of each is the difference between a sector and a triangle. Therefore, $\frac{\pi a R^2}{360} - \frac{1}{2} R^2 \sin a + \frac{\pi x r^2}{360}$ $- \frac{1}{2} r^2 \sin x = 2/5\, \pi r^2$, and since R sin $\frac{1}{2} a$ = half the common chord = $r \sin \frac{1}{2} x$, $R^2 : r^2 = \sin^2 \frac{1}{2} x : \sin^2 \frac{1}{2} a$. Simplifying and writing $\frac{1}{2}(1 - \cos x)$ for $\sin^2 \frac{1}{2} x$ we have $R^2 : r^2 = 180 \sin x - \pi(x - 144) : \pi a - 180 \sin a$ and $R^2 : r^2 = 1 - \cos x : 2 \sin^2 \frac{1}{2} a$.

Equating $\frac{180 \sin x - \pi(x - 144)}{\pi a - 180 \sin a} = \frac{1 - \cos x}{2 \sin^2 \frac{1}{2} a}$, putting for π, a, $\sin a$, and $\sin^2 \frac{1}{2} a$ their numerical values, and using logarithms in parentheses to represent the coefficients, (1.7219819) $\sin x$ + (1.1489865) $\cos x$ — (9.9638593) $(x - 144) = 14.092449$. Whence $x = 147° 1' 22.8''$, $r = 0.3990956$ R, $\pi r^2 = 0.5003845 R^2$, and $2/5\, \pi r^2 = 0.2001538 R^2$, the last of which is the same as the area of the space common to the two circles as required.

II. *Solution by E. L. Brown, M.A., Denver, Colo.*

Let C and O be the centers of the given and required circles, radii R and r respectively.

$$\text{Let ABC} = \frac{\pi}{4}, \ \text{AOE} = \theta, \ \text{ACO} = \frac{\pi}{8}.$$



From triangle ACO, $r = \dfrac{R \sin \frac{\pi}{8}}{\sin \Theta}$. Area common to the two circles = sector OAMB + triangle AOB + segment AEBD = $\dfrac{(\pi - \Theta) R^2 \sin^2 \frac{\pi}{8}}{\sin^2 \Theta} + \dfrac{1}{2} r^2 \sin 2\Theta + \left(\dfrac{\pi R^2}{8} - \dfrac{1}{2} R^2 \sin \dfrac{\pi}{4}\right) = \dfrac{2}{5} \pi R^2$. Hence

$\dfrac{\pi - \Theta + \frac{1}{2} \sin 2\Theta}{\sin^2 \Theta} = 8.3139$. By trial we find $\Theta = .64172$ radians, and $r = (.6393+)\ R$.

50. *Proposed by Mary E. Barkley, Sacramento, Cal.*

Find the locus of all the points, the sum of the squares of the distances of any one of which from *three* given points is equivalent to a given square.

Solution by J. F. West, San Diego, Cal.

Let A, B, C be the given points, k the side of the given square; M the centroid of triangle ABC; E the mid-point of CM and D the mid-point of AB. Denote the sides AB, BC, AC by c, a, b respectively.

In triangle PAB, $\overline{PA}^2 + \overline{PB}^2 = \dfrac{c^2}{2} + 2\overline{PD}^2$

In triangle PMC, $\overline{PC}^2 = -\overline{PM}^2 + 2\overline{EM}^2 + 2\overline{PE}^2 \therefore \overline{PA}^2 + \overline{PB}^2 + \overline{PC}^2 = \dfrac{c^2}{2} - \overline{PM}^2 + 2\overline{EM}^2 + 2\overline{PE}^2 + 2\overline{PD}^2 = \dfrac{c^2}{2} - \overline{PM}^2 + 2\overline{EM}^2 + 2(\overline{PE}^2 + \overline{PD}^2) = \dfrac{c^2}{2} - \overline{PM}^2 + 2\overline{EM}^2 + 2(2\overline{EM}^2 + 2\overline{PM}^2) = \dfrac{c^2}{2} + 3\overline{PM}^2 + 6\overline{EM}^2 = \dfrac{c^2}{2} + 3\overline{PM}^2 + \dfrac{2}{3}\overline{CD}^2 = \dfrac{c^2}{2} + 3\overline{PM}^2 + \dfrac{2}{3}\left(\dfrac{a^2}{2} + \dfrac{b^2}{2} - \dfrac{c^2}{4}\right) = 3\overline{PM}^2 + \dfrac{1}{3}(a^2 + b^2 + c^2) = k^2$.

The locus of P is therefore a circumference whose center is the centroid of the triangle ABC and whose radius is

$$\frac{1}{3}\sqrt{3k^2 - (a^2 + b^2 + c^2)}.$$

[NOTE.—This problem may be solved in the following neat manner: On page 404 of Volume VI, June, 1906, it was shown that "if P be the point of intersection of the medians of a triangle ABC, and Q any other point in the plane, then $\overline{QA}^2 + \overline{QB}^2 + \overline{QC}^2 = \overline{PA}^2 + \overline{PB}^2 + \overline{PC}^2 + 3\overline{PQ}^2$." In the present problem, PA, PB, PC and $\overline{QA}^2 + \overline{QB}^2 + \overline{QC}^2$ are given quantities, therefore the required locus is a circle whose center is the point P.—EDITOR.]

TRIGONOMETRY.

51. *Proposed by T. M. Blakslee, Ph.D., Ames, Iowa.*

If $X + Y + Z = 180°$, $\tan X + \tan Y + \tan Z = \tan X \tan Y \tan Z$.

Use this to find the radius of a circle inscribed in a triangle in terms of the sides.

Solution by Proposer.

If we join the center with the three vertices and contact points, we thus form three pairs of equal angles about this point. Their tangents are $\frac{s-a}{r}$, $\frac{s-b}{r}$ and $\frac{s-c}{r}$. Since the sum of these tangents is to be equal to their product, $\frac{s}{r}=\frac{(s-a)(s-b)(s-c)}{r^3}$, $\therefore r=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$

APPLIED MATHEMATICS.

52. *Proposed by Charles H. Smith, M.E., Chicago, Ill.*

A car trust is to be formed by the different companies now making the separate parts, pooling their interests; the capital is to be one million dollars. The trust is to pay the different firms by issuing 60 notes payable one each month for 60 months. The face value of each of the notes, including interest at six per cent, is to be the same. What is the face value?

Solution by H. E. Trefethen, Kent's Hill, Me.

There are as many solutions of this problem as there are methods of computing interest on notes when payments in part have been made. Whatever the method it should be noted that any payment must exceed the interest on the principal for the preceding interval in order to cancel the debt as required. Of the two methods in general use the United States Supreme Court rule compounds the interest, while the Mercantile rule allows only simple interest on both the principal and the payments.

Put \$1,000,000 $= k$, $60 = n$, and let x be the face of each of the equal notes.

1st. By the U. S. Rule, put $1.005 = r$, and $rk - x =$ sum left after first payment. In like manner $kr^2 - xr - x$, $kr^3 - xr^2 - xr - x$, $kr^n - xr^{n-1} - xr^{n-2} - \dots - xr - x =$ sums left after the second, third and nth payments, which last must equal zero. Whence $x = \frac{kr^n}{1+r+r^2+\dots+r^{n-1}}$. This denominator is a geometrical series an its sum is $\frac{r^n-1}{r-1}$. Therefore $x=\frac{k(r-1)r^n}{r^n-1}$. Replacing the numerical values and solving by use of logarithms, $x =$ \$19,332.52.

The same result is obtained by the Compound Interest method, compounding the interest every month. For the amounts of the several notes at compound interest from the time each is paid to the end of the sixty months taken together should equal the amount of the capital at the same rate of interest for the sixty months and we have, since the last payment with no interest is x, $x(1+r+r^2+r^3+\dots\dots+r^{n-1})=kr^n$.

Hence $x=\frac{k(r-1)r^n}{r^n-1}=$ \$19,322 52, as above.

2d. By the Mercantile Rule, putting $.005 = r$, the amount of the principal for the whole time $= k(1 + nr)$. The amount of the first payment (for 59 months) is $x + x(n - 1)r$, and in like manner $x + x(n - 2)r$, $x + x(n - 3)r$, etc., to the nth and last payment, which with no interest is x. The sum of these payments, which must equal the amount of the principal, is $nx + rx[(n - 1) + (n - 2) + (n - 3) + \ldots\ldots + 1]$. The series in brackets is arithmetical and its sum is $\frac{1}{2}(n - 1)n$. Hence $nx + \frac{1}{2} rx(n - 1)n = k(1 + nr)$ and

$$x = \frac{k(1 + nr)}{n + \frac{1}{2} nr(n - 1)} = \$18,881.63.$$

3d. Another method, which is well suited to this problem and accords with the Connecticut Rule, and also with the Vermont and New Hampshire Rules in the case of interest *annually*, is to find the amount of the debt and of each of the payments to the end of each year. Put $1.06 = r$. Twelve payments a year are made each $= x$. The first is on interest for 11 months, the second for 10, etc., to the last, which paid at the end of the year has no interest. Therefore the sum of the amounts of the 12 payments is $12x$ plus the interest of x dollars for 66 months or $.33x$. $12x + .33x = 12.33x$. Subtracting this from the amount of the principal for the year, leaves as a principal for the second year $kr - 12.33x$. The second remainder is $kr^2 - 12.33xr - 12.33x$ and the fifth, $kr^5 - 12.33xr^4 - 12.33xr^3 - \ldots\ldots - 12.33x$.

The last remainder must be zero. Hence $12.33x = \dfrac{kr^5}{r^4 + r^3 + r^2 + r + 1}$

$= \dfrac{k(r - 1)r^5}{r^5 - 1}$ and $x = \$19,253.55$.

CREDIT FOR SOLUTIONS RECEIVED.

Algebra 40. J. W. Ellison, I. L. Winckler, H. E. Trefethen.

Algebra 44. H. E. Trefethen.

Algebra 48. R. P. Harker, H. E. Trefethen, G. J. Van Buren, Franklin T. Jones, A. J. Wile, I. L. Winckler, E. L. Brown.

Geometry 45. J. R. Parker, H. E. Trefethen, E. C. Thayer.

Geometry 49. H. E. Trefethen, E. L. Brown.

Geometry 50. J. F. West, H. E. Trefethen, E. L. Brown, I. L. Winckler.

Trigonometry 51. T. M. Blakslee, H. E. Trefethen, I. L. Winckler, E. L. Brown.

Applied Mathematics 43. H. E. Trefethen.

Applied Mathematics 47. H. E. Trefethen.

Applied Mathematics 52. H. E. Trefethen, E. L. Brown.

Total number of solutions, 28.

PROBLEMS FOR SOLUTION.

ALGEBRA.

58. *Proposed by H. C. Whitaker, Ph.D., Philadelphia, Pa.*

The money that will pay the wages of Tom Jones for 61¼ days

will pay the wages of Harry Smith for $81\frac{2}{3}$ days. For how many days will the same sum pay the wages of the two men?

59. *Proposed by H. C. Whitaker, Ph.D., Philadelphia, Pa.*

At a certain time, a train overtakes a man and ten seconds thereafter passes him. Twenty minutes after passing this man, the train meets another man and in nine seconds thereafter passes the second man. Counting from the time that the train passed the second man, how soon will the two men meet?

GEOMETRY.

60. *Proposed by I. L. Winckler, Cleveland, O.*

Find a point in a given straight line from which, if tangents are drawn to two given circles, they will make equal angles with the given line. (From Chauvenet's Geometry.)

61. *Proposed by Russell P. Harker, Parker, Ind.*

Given the base $2b$ of a triangle and α, the difference of the base angles. Find the equation of the locus of the vertex. (From Bailey and Wood's Analytic Geometry.)

MISCELLANEOUS.

62. *Proposed by John W. Scoville, Syracuse, N. Y.*

A prison consists of 36 cells arranged like the squares of a chess board. There are doors between all adjoining cells. A prisoner in one of the corner cells is told that he can have his freedom, if he can get into the diagonally opposite corner cell, by passing through each of the cells once and only once. Can the prisoner win his freedom?

DEPTH OF DEATH VALLEY, CAL.

LOWEST POINT IN UNITED STATES.

The United States Geological Survey has just completed a line of spirit levels through Death Valley, Cal., and much to the surprise of everyone familiar with the region has ascertained that the depth of that area is not so great as was supposed. The final computations of the results have not yet been made, but the preliminary figures give for the lowest point a depth of 276 feet below sea level. Bennetts Well, which is near this point, is 266 feet below sea level. These figures may be altered by two or three feet when the final computations are made, but they are probably not more than three feet in error. The Geological Survey now has elevation marks on the highest and lowest points of dry land in the United States.

It is a strange coincidence that these two extremes are both in southern California and only seventy-five miles apart. Mount Whitney is a foot or two over 14,500 feet above sea level, while Death Valley, as above stated, is 276 feet below. Before the Salton Sink, also in southern California, was flooded by the Colorado River, it contained the lowest point of dry land in the United States, a spot 287 feet below sea level.

Previous estimates of the depth of Death Valley based on barometer readings gave for the lowest point figures varying from 250 to 450 feet below sea level. The level line of the Geological Survey is believed to be the first accurate determination of elevations in that locality that has ever been made.

DEPARTMENT OF SCIENCE QUESTIONS.

FRANKLIN T. JONES,

University School, Cleveland, O.

This department is designed to serve as a medium for the exchange of ideas on questions and questioning in the sciences. Questions will be printed from various sources—college entrance examinations, text-books, etc. Comment is invited. Suggestions and criticisms as to character, adaptability and usefulness are desired. Readers of this journal are invited to propose questions and problems which will be of general interest, or of a type which will be useful in the class-room. It is not expected that questions which will not be useful to pupils will be frequently printed.

Since the majority of the questions will be of a comparatively simple character, solutions and answers will not be published unless specifically asked for. Teaching suggestions are wanted.

Address all communications to the editor of the department.

9. In a test of the new electrical locomotive for the N. Y. C. R. R. the indicator showed a speed of thirty miles per hour in sixty-three seconds from the time the controller was put on the first notch. What was the average acceleration?

As the controller was thrown over, the speed increased at the rate of five miles per hour every thirty seconds. How long did it take to reach the maximum speed of seventy miles per hour? (News Item.)

10. If a force of six hundred thousand dynes is resolved into mutually perpendicular components making equal angles with the given force, find the value of the components. (Sheffield.)

11. (*a*) If a rifle-bullet were shot directly upward with a velocity of one thousand feet per second, how far would it rise in a vacuum?

(*b*) If the bullet weighs one ounce, how great will be the potential energy at the greatest height? (Name the unit of energy). (Harvard.)

12. Which requires the greater amount of heat, to raise one thousand grams of water from ninety-seven degrees to ninety-nine degrees Centigrade, or to raise the same amount of water from ninety-nine degrees to one hundred and one degrees Centigrade, under ordinary atmospheric pressure? Why?

Change eighty degrees Fahrenheit to Centigrade; one hundred eighty degrees Centigrade to Fahrenheit, and to absolute scale. (Princeton.)

13. The two branches of a divided circuit have resistances of three ohms and five ohms respectively. The total current in the circuit is twenty-four amperes. What is the current in each branch? Explain. (Board.)

14. Describe the phenomena of total reflection. (Sheffield.)

15. Ten grams of crystallized sodium sulphate were found by experiment to contain 4.70 grams of water. How many molecules of water of crystallization does the crystallized salt contain? (Harvard.)

16. Give Avogadro's law. If one hundred and five million molecules of hydrogen burn in oxygen, how many molecules of water are formed? (Sheffield.)

17. What evidence have we that matter is indestructible? If we cannot create matter, how are we to account for the fact that the products of combustion of a candle weigh more than the original candle? (Princeton.)

18. What is a flame? What are the conditions essential to (*a*) its luminosity, (*b*) its maximum temperature?

Make a sketch of a Bunsen burner and its flame, illustrating in detail and explaining the parts of each. (Board.)

19. What substances are contained in ordinary gun-powder? Explain why it explodes. (Sheffield.)

20. Explain how the human body obtains energy from the sun. (Harvard.)

MATHEMATICAL ANNOUNCEMENTS.

The report of the committee on the Teaching of Geometry which was presented at the last meeting of the Association is now in press. Copies of this report will be mailed soon to all members of the Mathematics Section of the Association. Copies can be obtained fro those outside of the Association by enclosing a two-cent stamp to the Secretary of the Mathematics Section, Miss Mabel Sykes, 438 57th St., Chicago.

This report is important to teachers of high school mathematics in many ways. It is replete with suggestion as to ways of handling geometry. It shows that a careful study of elementary geometry by strong people is productive of great fruit, that many things may be improved in numerous ways by almost any teacher who cares to take the trouble; it gives some standards of judging of good teaching, and is valuable in so many ways that no teacher of geometry can afford to be without it. No one can read this report carefully and leave it with the impression that all has been done that can be done for the improvement of geometry as a school subject.

The council of the *Association of Teachers of Mathematics in the Middle States and Maryland* has authorized the organization of a self-governing section of that association in the vicinity of Rochester, N. Y. A preliminary meeting to organize the section occurred at the University of Rochester on February 23. The friends of the teaching of mathematics will wish their New York colleagues all success in this enterprise. Incidentally, we remark another evidence of the fact that a *renaissance* in the teaching of mathematics is on among us.

Professor H. E. Slaught's name appears on the title page of the *American Mathematical Monthly* of Springfield, Mo., and this well-known journal now announces its publication UNDER THE AUSPICES OF THE UNIVERSITY OF CHICAGO.

METRIC VOICE FROM AUSTRALIA.

EXTRACTS FROM THE "MELBOURNE AGE," NOVEMBER 27, 1906.

The prospects of a reform in our cumbersome British weights and measures and coinage are looking brighter. In the new British Parliament, as in the last, a majority of the members are in favor of a sweeping change. Even in the Transvaal, amid the turmoil of getting the country straight after the war, a Commission was recently appointed to take evidence and report upon improvements in weights and measures. From its report it appears that public opinion in the Transvaal is strongly in favor of a bold adoption of the metric system. But the South African Commission gives practical proof of its earnestness and sagacity by recommending that the use of the metric system be made compulsory in land surveying and the sale of drugs. The reason for selecting these two departments by way of a commencement is obvious. Almost all the important land transactions in a country are local. An inappreciable fraction of the transactions in land either in Australia or South Africa is carried on with Britain. Hence in adopting an improved method of expressing land measurements quite independently of the old country, the self-governing parts of the Empire introduce no element of complication into their trade relations with Britain. In the drug trade the measurements are in the hands of expert pharmacists, to whom a change from the absurd apothecaries' measures would present no difficulty, but a welcome relief. Moreover, the British system of measures for land is the most fantastic of all the displays of unreason afforded by our weights and measures. In this twentieth century the table for lengths sounds almost crazy in its eccentricities—12 inches 1 foot, 3 feet 1 yard, 5½ yards 1 rod, 25 links 1 rod, 22 yards or 4 rods 1 chain, 10 chains 1 furlong, 8 furlongs 1 mile. In the surface table the absurdity culminates in the relations 144 square inches 1 square foot, 9 square feet 1 square yard, 30¼ square yards 1 perch, 40 perches 1 rood, 4 roods 1 acre, and 640 acres 1 square mile. The bare recital of these tables proves them to be freakish barbarisms and anachronisms that are quite intolerable. The Transvaal Commission's recommendation is wise and sound that these absurdities should be swept away by making the metric system compulsory in land surveying.

It is estimated that it costs £1,000,000 a year to teach the British weights and measures to British children, and that it wastes a whole year out of the schooling of each child. It is simply impossible to estimate with any accuracy the loss in which adults are involved by our inconvenient weights and measures. With the metric system and decimal money the compound rules of arithmetic disappear. As a compound rule sum takes about five times as long as the corresponding simple rule sum, the total waste of time involved in the retention of our antiquated system is huge. Moreover, in all the Empire's foreign business with the fifty metric nations there is a steady loss of Britain's prestige going on.

One little fact shows that the British system is doomed, and should be got rid of as soon as possible. New industries even in England itself

adopt the metric system. For instance, in the manufacture of motor cycles and motor cars the sizes are well worked out with the metre divided into tenths, hundredths, and thousandths. In the very home of the engineering trade it is most significant that the newest branch should take to the metric system as a necessity. The universal change has got to come, and the less the delay over it the better. The total loss under the present antique system is too grievous to be borne any longer.

THE INCH ABSURDITY.

The British Weights and Measures Association was created to combat the rapidly growing metric sentiment in England. Its aim apparently is to perpetuate for all time and to introduce into all countries the ridiculous English weights and measures, with variations. Realizing the weakness of a so-called system that has no systematic co-relation of its parts, lineal, volumetric, gravimetric and specific, it has at length proposed to decimalize the inch, making that the lineal unit, the square inch the surface unit, and the cubic inch the unit of volume. For weight it would introduce a new grain, a new ounce and a new pound, thus making confusion more confounded.

Mr. S. Jackson has written and the Decimal Association has published an eight-page leaflet to show the absurdity of such an attempt. The weight of a cubic inch of water at standard temperature and pressure is 252.258 grains. Now it is proposed to change the value of the grain so that a cubic inch of water shall weigh 250 grains. The folly of such an unscientific change is shown—in fact it comes pretty near being self-evident to the average thinking man—(1) by the practica. impossibility of making people use it; (2) by the great confusion that would arise, for there would then be two sorts of grains, two of ounces, two of pounds, the old and the new, besides the metric system. Moreover the new weights would be useless outside of Britain, even in her colonies and in the United States; (3) by the absurdity of supposing that the meter can ever be replaced by the inch as a universal unit.

The fact that some fifty countries have adopted the metric system and not one has gone back to its former units and usage, and that no country has ever adopted, or expressed the slightest desire to adopt, the English, outside of its own colonies, which are themselves now turning away from it, as is England herself, justifies Mr. Johnson's contention. (4) The misnomer of calling this a decimal system, and making people believe that it is.

Assuming the possibility of so radical a change as the one contemplated by the British Weights and Measures Association, we must see that the new system is in no sense a perpetuation of the present one, but rather a new one founded on the old inch and a new grain. Those who have a reverence for preserving the antique British weights should reflect that the proposed change is almost as iconoclastic as the metric. Where could it be better than the metric, from any point of view? On the other hand, let us ask: Is it not inferior to the metric in every point?

R. P. W.

A PRELIMINARY SCIENCE COURSE.

To the Editor:

DEAR SIR: In the February number of your journal I note a communication from Professor G. W. Stewart stating his objections to the proposed general science course mentioned in Circular IV of "The New Movement among Physics Teachers."

The arguments there presented are logical and well put, and certainly cover one view of the case in an able manner; but to one who has had experience with an elementary and preparatory course in science, and has had opportunity to observe its effects, the stand taken in the communication seems untenable. I desire therefore to submit to the readers of this journal some reasons why I consider such a course highly desirable and profitable.

However, before so doing, it will be necessary to reach some agreement or at least to suggest some outline as to what such a proposed course shall comprise. Without any desire or expectation of anticipating the report of the commission on this proposed course, but rather on the supposition that suggestions and opinions upon this matter are invited I beg to offer the following plan for such a course:

I believe the course should consist in the main of elementary physics with a little of chemistry, but nothing of any other science as such. It should especially emphasize those principles of physics which find application in physical geography and the other sciences. At this point permit me to suggest that the proposed course would not be preparatory to physics alone, but to all the science work of the secondary school. Since physics is the most general science and contains the first principles of science, it is plain that some of it at least should be met by the pupil before his junior or senior year.

Again, the course must be distinctly experimental and inductive, and it should be taught in the laboratory where the pupils are in the midst of things and are thrown largely upon their powers of observation. As the commission suggests, the work must be qualitative, and it must of necessity be made up chiefly of topics found in the high school text-book of physics, but treated in a more elementary way, with greater stress given to illustrations of principles. While the course might be made a sort of "laboratory nature study" yet let us not err by making it too elementary; and let us not underestimate the power of first year pupils to comprehend, especially when the work is so largely experimental.

The instructors must be laboratory teachers skilled in the use of apparatus; and they must give the class their best efforts, making the experiments speak for themselves without apology from them.

To be more explicit, the following topics from physics and chemistry are suggested as an outline for a course of half a year:

1. Matter and its physical properties (only such as can be illustrated or are familiar to the pupils).
2. Forces—adhesion, cohesion, capillarity, surface tension, centrifugal force (gyroscope), gravitation and gravity.

3. Chemical phenomena and processes—chemical change, combustion, solution, filtration, evaporation, elements and compounds (the latter to be exhibited so far as possible).

4. Study of the gases—oxygen, hydrogen, nitrogen, carbon-dioxide, their uses and relation to plants and animals.

5. Water—formed of gases, decomposition, solutions, properties, natural waters, filtration and distillation (filtration plants), mineral waters.

6. Atmospheric pressure, with particular application to meteorology—the barometer.

7. Fluid pressure, siphons, pumps, springs, artesian wells, buoyancy of water and air, balloons, exploration of the upper air.

8. Heat—sources, thermometers, expansion, irregular expansion of water and consequences, force of expansion, fusion, evaporation, boiling point.

9. Effect of large bodies of water on temperature, land and sea breezes, clouds, artificial cold.

10. Transmission of heat—conduction, convection, radiation, convection in nature, high and low pressure areas, currents.

11. Hygrometry—dew point, relative humidity, rain, fog, clouds, weather bureau and instruments, use by pupils of instruments, observations, curves of temperature, humidity and pressure.

12. Magnetism and electricity—needle, earth, compass, frictional electricity, lightning.

13. Light—shadows, eclipses, reflection, refraction, prismatic colors, rainbow.

No details of a general science course have yet been given out by the commission, but any such course could not differ very radically from the above if it were to be a preparatory course. It might be argued that the course would intrude too far into the regular physics work, bu all depends upon how it is taught and what is taught under the topics. The same subjects may be used in a grammar school and a college, but there will be a difference in the treatment of the subject matter.

Physics is a large field, and the usual high school course should really have more than a year of time. A preliminary course could do much to relieve the tension in physics and in this way help solve the present problem. Mr. Stewart feels that the proposed course will make physics even more technical and difficult, the very condition that we should be striving against, but with greater preparation on the part of the student a more technical course would not seem so. There can be no great objection to a technical course so long as the student is prepared for it. Is there any danger of the pupil's learning too much?

It is true that there will be some repetition, but this is necessary for thoroughness and especially so when the courses are two or three years apart. It is generally admitted that people are more interested in what they already know something about, and if this is true, it is not easy to see how interest in physics can be lessened by reason of having some previous knowledge of it. It is the testimony of those who

have observed the working of such a course that it has increased the interest and enlisted more students in the science courses.

A first year course similar to that outlined is in use in a number of high schools, and so far as I am able to learn it is in every case looked upon with favor. In the Columbus high schools, the elementary science work at present occupies the first three months of the first year, followed by physical geography the rest of the year. While this science course has not been in use long enough for its full effects to be observed, yet it is considered indispensable, and more importance is attached to it every year.

In conclusion I beg to give a summary of the reasons for the course in question:

1. The knowledge to be acquired by such a course should be the possession of every pupil before he reaches his junior or senior year.

2. The course is needed as a preparation for all the high school sciences.

3. Fewer than one third the pupils entering the high school reach the junior and senior years, and without the course two thirds are deprived of any knowledge of physical science.

4. Such a course enhances the interest in science subjects.

5. The training in cultivating the power of observation, and practice in recording the results.

6. Some of the subjects of the usual physics course, such as properties of matter, pumps and siphons, could be largely omitted, thus relieving the pressure and allowing time for laboratory work.

7. Such a course affords an opportunity for the teacher to give valuable qualitative experiments, which the pupil should know, that he cannot find time to use in the physics course.

8. The laboratories are the most expensive departments of the school, and these courses are enjoyed by too small a number of the pupils.

R. O. Austin,
Central High School, Columbus, O.

UNITED STATES GEOGRAPHIC BOARD.

DECISIONS.

The following important decisions relating to geographic names and their application were made by the United States Geographic Board on February 6, 1907. In reaching these decisions the Board has obtained the advice of many of the foremost American geographers and geologists, and the decisions here given are, in nearly all cases, the result of a concensus of opinion among the gentlemen consulted:

Cordilleras; the entire western mountain system of North America.

Rocky Mountains; the ranges of Montana, Idaho, Wyoming, Colorado, New Mexico, and western Texas.

Plateau Region; the plateaus of Colorado River and its branches, limited on the east by the Rocky Mountains, on the west by the Wasatch Range, and extending from the southern end of the Wasatch southwestward, southeastward, and eastward to the eastern boundary of Arizona, following the escarpment of the Colorado Plateau, and including on the north the Green River Basin.

Basin Ranges; all those lying between the Plateau Region on the east, the Sierra Nevada and Cascade Range on the west, and the Blue Mountains of Oregon on the north, including the Wasatch and associated ranges.

Pacific Ranges; the Cascade Range, the Sierra Nevada, and the coast ranges collectively.

Sierra Nevada; limited on the north by the gap south of Lassen Peak, and on the south by Tehachapi Pass.

Cascade Range; limited on the south by the gap south of Lassen Peak and extending northward into British Columbia.

Coast Ranges; extend northward into Canada and southward into Lower California, and include all mountains west of Puget Sound and the Willamette, Sacramento, and San Joaquin valleys, and southwest of Mohave Desert.

Bitterroot Range; extends from Clarks Fork on the northwest to Monida, the crossing of the Oregon Short Line on the southeast, including all mountain spurs.

Mission Range; range east and southeast of Flathead Lake, Montana.

Wasatch Range; includes on the north the Bear River Range, extending to the bend of Bear River at Soda Springs, Idaho, and on the south extends to the mouth of San Pete River near Gunnison, Utah.

San Juan Mountains; include all the mountains of southwest Colorado south of Gunnison River, west of San Luis Valley, and east of the Rio Grande Southern Railroad.

Sacramento Mountains; include those groups known as Jicarilla, Sierra Blanca, Sacramento, and Guadalupe.

Salmon River Mountains; include the group in central Idaho lying south of the main Salmon River, west of Lemhi River, north of Snake River, and east of the valley of Weiser River.

Blue Mountains; include all the mountains of northeastern Oregon with the exception of the Wallowa Mountains, and extend into Washington.

Sangre de Cristo Range; extends from Poncha Pass, Colorado, to the neighborhood of Santa Fe, N. Mex., thus including the southern portion, locally known as the Culebra Range.

Front Range; includes on the north the Laramie Range as far as the crossing of the North Platte, and on the south includes the Pikes Peak Group.

Appalachian System; includes all the eastern mountains of the United States from Alabama to northern Maine.

Blue Ridge; includes the ridge extending from a few miles north of Harpers Ferry to northern Georgia.

Appalachian Plateau; includes the entire plateau forming the western member of the Appalachian System, known in the north as the Allegheny Plateau and in the south as the Cumberland Plateau.

Ozark Plateau; the plateau in northwestern Arkansas and southern Missouri.

Ouachita Mountains; the ridges of western Arkansas south of the Arkansas River, Indian Territory, and Oklahoma.

THE NEW MOVEMENT AMONG PHYSICS TEACHERS.—CIRCULAR V.

In response to the suggestions and questions in circulars III and IV, 164 answers have been received. These have come from 105 secondary schools, 7 normal schools, and 52 colleges. Of these 164 answers, 43 merely expressed general approval of the work, and 24 more asked to have the remainder of the syllabus sent to them when it was ready. There were 42 who answered all of the four questions at the end of circular IV. Of the remaining 55 letters, each discussed one or more of the various points suggested.

I. Taking up first the theses in circular III, 14 approved of them in toto just as they stand, and 18 others approved of them in general, but each made a few specific suggestions as to desirable changes. In the light of the suggestions that have been received, the theses have been reworded, an eleventh added, and they are now again submitted to the teachers for criticism and suggestion in the following form:

1. The subject-matter of the present elementary course in physics must be reduced to two-thirds of its present amount, unless the time allowed for covering it be increased to one and one-half years.

2. If the subject-matter is reduced, those topics that have the least bearing on the student's life and on the problems likely to occur to him spontaneously from his own experiences should be eliminated first. The better established portions of the subject should have precedence over the more recent unproved speculations; on the ground that, in the limited time, it is better to teach those things that will probably be still believed when the youngster is grown up.

3. In the elementary course the method of presentation is far more important than the amount of subject matter learned. This method should be so framed that the emphasis is laid on the development of habits of scientific thought, rather than on the mastery of subject matter. Hence it is better to present a few topics in such a manner that they are powerful examples of the method by which science obtains its results, than to try to teach a large number of more or less scattered facts and theories in such a way that they can only be committed to memory.

4. In applying practically the principle of thesis 3, it is important that definitions be justified before they are introduced. In order to do this, the concepts with which a definition deals should be built up in the student's mind by a discussion of familiar experiences, and he should be led to see that there exists among the concepts a relation that admits of definition, before the definition is stated. A definition that has been so introduced will be appreciated as a convenience, and every one that is not so appreciated had better be omitted from the required work.

5. In like manner, it is generally not advisable to state a law, until the concepts and relations with which it deals have been implanted in the student's mind by a discussion of common experiences and of simple qualitative demonstrational experiments. In other words, the student will generally not appreciate the law unless he be given

an intuitive and qualitative perception of the relations summarized by the law before the law is stated; i. e., the law should be to him a hypothesis before it becomes a law.

6. The student should be made to see clearly that the laboratory experiments furnish the means of converting hypotheses into laws. He should also be made to see that the apparatus is not the law, that it is not necessary to remember the details of the apparatus in order to appreciate the law, and that the exemplifications of the law are not confined to the apparatus.

7. The student should be made to comprehend that every law is a tested hypothesis, and that the tests are always subject to some error, so that the statement of the law is always a statement of what we believe to be true in an ideal case. He should understand that the measurements by which a law is said to be established give results which approach more and more nearly to the law, the more carefully the measurements are made, and the more completely the disturbing effects are eliminated. He should also be shown that in every practical case the law is not verified unless allowance is made for friction, air resistance, etc.

8. In the laboratory work it is often more profitable to place the emphasis on the determination of efficiency rather than on the verification of laws. This sort of work shows clearly the practical use of the experiment, prevents false notions of the mechanical advantage of machines, helps to make clear the importance of this concept in the world's work, and tends to develop in the student a hearty respect for the value of quantitative knowledge.

9. As few units as possible should be employed, and they should be introduced only when a necessity for their use appears; i. e., they should be justified in advance as in the case of definitions and laws. By this thesis the more abstract units like the dyne and the erg would no longer be required in the elementary work.

10. Examinations and quizzes should be framed to test the student's comprehension of and ability to use the more important principles of physics. The questions should not ask for mere statements of the laws from memory, unless they also ask for either the arguments by which the laws are established, or for information concerning the way in which the principle is applied in daily life. These questions should not contain complicated arithmetical puzzles of the sort that never occur in practical work, but should contain simple problems which deal with immediate concrete applications of the principles, and which are of the kind likely to be met with outside of the class-room or laboratory. They should not demand descriptions of laboratory apparatus nor of facts which have no immediate bearing on the general principles required by the syllabus.

11. The distinction between the real facts, which are matters of definite knowledge, and the supposed facts, which are derived from pure speculation, should be kept clear in the student's mind. For example, he should know that he is speculating when he explains the properties of gases in terms of the hypothetical molecules of the gas,

and that he is dealing with definite knowledge when he describes those properties in knowable factors like volume, pressure, density, and temperature. He should be trained to know what the things about him can do, rather than to think that he knows why they do it, because he has learned to repeat the beliefs of others concerning the modus operandi.

II. The following comments were called forth by the preamble to the definition of the unit in circular IV: Interest is not a necessary forerunner of knowledge (1). The ideas of force and of moments are better than that of energy as the central concepts (1). The scientific method is confused with induction (1). Too much historical physics is bad; on review, the student can be made to look up encyclopedias (1). Much more history is needed (4). On this point it seems desirable to quote a few sentences from one of the letters received, as follows: "It has long seemed to me that one serious defect of the teaching of science in our schools and colleges has been the apparent isolation of the subject-matter taught from the ordinary concerns of life. One way to remedy this defect is to provide carefully prepared courses in the history of each science and to teach these courses in connection with the courses in science themselves. In this way, the part that science has played in the development of our civilization, and all that it means to the world today, would be brought home to the pupils. Such a result could hardly fail to be a stimulus to the patient and serious study of the science of physics."

III. Concerning the definition of the unit, the following criticisms and suggestions were received:

One demands that the required time of the course be increased to 280 periods, while 10 declare 240 periods too much. Four want to see the requirement of lecture demonstration work emphasized more, four want all the laboratory experiments written up, and four more object to seeing the attempted determination of physical constants in the laboratory discouraged. Opinions differ as to the number of laboratory experiments to be required. Four want 40; one, 36; two think 35 enough; seven unite on thirty; and three ask for a requirement of only 20.

Some emphatic opinions were sent in with regard to the question of allowing part of the required laboratory work to be qualitative. In six letters the permission of qualitative experiments is hailed with delight, while in four others it is condemned in a no less decided way. One suggests that physics be taught in the natural history style, leaving the mathematical parts to the mathematics teacher, and another insists that the greatest value of the physics course lies in making clear the worth of definite quantitative knowledge.

In reply to question 1, namely, does the definition of the unit seem to be what is needed, 34 out of 42 answered in the affirmative, and eight made some suggestions as to changes.

Question 4, namely, is the plan of starring topics more satisfactory than that of a list of experiments? received 35 affirmative votes, and

two negative. Two stated that it was a matter of indifference, two suggested using both, and one wished to have neither.

In consideration of the fact that the definition specifies only the minimum amount of work that will be accepted as a unit, and in the light of the suggestions received, this definition has been slightly altered and is again submitted for criticism as follows:

REVISED DEFINITION OF THE UNIT.

1. The unit in physics consists of at least two hundred periods of forty-five minutes each (= 150 hours) of assigned work. Two periods of laboratory work count as one of assigned work.

2. The work shall consist of three closely related parts, namely, class work, lecture demonstration work, and laboratory work. At least one-third of the time shall be devoted to the laboratory work.

3. It is very essential that double periods be arranged for the laboratory work.

4. The class work shall include the study of at least one standard text.

5. In the laboratory, each student shall perform at least thirty individual experiments, and keep a careful note book record of them. Twenty of these experiments must be quantitative; each of these must illustrate an important physical principle which is one of the starred topics in the syllabus, and no two must illustrate the same principle.

6. In the class work the student must be drilled to an understanding of the use of the general principles which make up the required syllabus. He must be able to apply these principles intelligently to the solution of simple, practical, concrete problems.

7. Examinations will be framed to test the student's understanding of and ability to use the general principles in the required syllabus, as indicated in paragraph 6.

8. The teacher is not expected to follow the order of topics in the required syllabus unless he wishes to do so.

In explanation of the term required syllabus, the commission voted at its meeting in New York to separate the proposed syllabus into two parts, one giving merely the principles required as a minimum amount of work, and the other an expanded and suggestive syllabus similar to that submitted in circular IV. It is to the former of these that the term required syllabus refers.

In reply to question 2 of circular IV, namely, is the form of the syllabus satisfactory? one negative and thirty-five affirmative votes were received. Five objected to having any syllabus at all.

Question 3, namely, do you wish to have either the choice of subject matter or its arrangement in the syllabus altered? received 24 ayes and 18 nays. Those who want changes made have sent in a large number of valuable suggestions, and these will be submitted to the commission as soon as possible. In consideration of the marked differences of opinion that have appeared respecting the syllabus, the commission has decided to withold the publication of the remainder of it, awaiting the results of a discussion now in progress in the com-

mission. Those who have sent requests for the new syllabus will receive it as soon as it is ready for distribution.

IV. The suggestion in circular IV that there be introduced into the first year of the high school curriculum an elementary course in general science, including some physics, has called forth some decided opinions. Three oppose the idea on the ground that it would take the edge off the real course in physics which comes later, and four others declare the proposition impractical because of the impossibility of getting the requisite time. Two claim that it has been tried in their schools and found harmful, and one thinks that such a course belongs in the grades. One says that it should be required physiography. Four express the conviction that such a course should be worked up, four report having tried it with marked success, and three hope that such a course will be introduced in order that all the pupils in secondary schools may get at least a taste of science. (This latter in consideration of the fact that about 50 per cent of those who enter the high schools drop out by the end of the second year.)

V. Since issuing the last circular, the commission has held two meetings, one in Chicago on November 30, and one in New York on December 29. At the first of these meetings ,the question of the influence of examining boards on the teaching of physics was discussed. This discussion is still in progress in the commission, and the results of it will be announced as soon as they have been reached.

At the New York meeting, two important matters were discussed; one, the form and content of the proposed syllabus; and the other, the nature of the problem before the commission. In regard to the first of these questions, the commission voted to prepare a double syllabus; one, to be very brief and to include only the principles required as a minimum of work for the unit; the other, an extended syllabus, intended to be suggestive, somewhat like that in circular IV. In the first syllabus the attitude of the teacher toward each principle may be defined by several leading questions. The commission is now at work on these outlines.

In regard to the second question, it was unanimously agreed that the commission stands for a maximum degree of freedom for the teacher: that its work lies in the direction of discussing and making clear the principles that may guide the teacher, and in supplying suggestions that may assist him in making his work stronger: that the teacher should be at liberty to apply the principles as outlined by the commission in the way he thinks best, and, in particular, he should be left in complete freedom in the choice of the tools with which he works. For these reasons, the commission agreed to exclude entirely from its discussions all questions relating to the merits of particular texts, manuals, or apparatus. It is important that everyone should clearly understand that the work of the commission lies in the direction of solving an important educational problem, and that this work is to remain wholly free from implication in any way with books or with apparatus.

VI. On counting up the letters received by the commission during the past year, it appears that suggestions and criticisms have been received from 418 different teachers. Of these 270 are in secondary schools, 113 in colleges, and 35 in normal schools. Every state in the Union has a representative among this number, excepting Delaware, Florida, Idaho, Arkansas, and Nevada. Yet the only conclusion that all can agree to draw from the summaries of the answers as printed in the various circulars is that we teachers are far from united on any one point. Some insist on the introduction into the laboratory of a large amount of qualitative work; others are equally insistent that this work should be all quantitative. Some want to emphasize the ideas of energy; others prefer to base the work on concepts of force. Some approve of making the course strongly inductive, and of trying to teach scientific method of thought; others declare this method useless, and insist on mastery of subject matter. Some declare that the course should be limited to pure science; others believe in making it more practical. Some wish to introduce more mathematics; others want to teach in the natural history style entirely, etc.

Under these condiions it seems fair to ask whether the reason for these wide diversities of opinion is not this: that we are trying to do too many things in the one year allotted to physics. If too much is expected of this one year's work, one teacher will emphasize one phase, another will cling to another, and there never can be even approximate agreement. It therefore seems plausible to adopt, as a working hypothesis which shall bring these discordant observations into harmony, the one just suggested, namely, that we are trying to accomplish too many different things in this one year. The aims of the course are at present too diffuse to be clearly grasped by anyone.

If we are willing to adopt this working hypothesis, a solution of the difficulty at once appears. This solution was suggested in one of the letters recently sent in and is briefly this: that the teaching of physics must not be crowded into one poor year in the secondary school, but must be done partly in the grades in connection with the nature study work, partly in a first year of general science in the high school, and partly in the special physics course in the later years of the high school, to be continued as far as desired in college. These successive bits must not overlap in such a way as to give the student the impression of going over the same ground in the same way, as is at present the case. It therefore seems clear that the commission cannot ever reach a satisfactory solution of the problem before it, until it has discussed the entire question of the education of the child in physics in all grades.

Numerous efforts at earlier work in physics have been made in various schools during the past few years; the first step in the investigation is, therefore to try to find out what work has already been done, and what success has been attained. You are therefore invited to contribute to this investigation by sending answers to any or to all of the following questions:

1. Has your school ever attempted to give a first year's work in general science? If so, what success was attained? What outline was used? What was the aim of the course? Where can the text or the outline used be obtained?

2. Do you think a well-coördinated four-year course in science is desirable in the high school, the first year to be required, the rest to be elective? This does not mean that there be a year in each of the separate sciences, but that at least the first year's work be in general science—a combination of several.

3. Can you suggest a series of steps in the method of presenting science—steps by which it would be possible to pass gradually from the nature study methods of the grades to the abstract methods of the college? In what years in the curriculum should the successive steps be taken? This amounts to asking for a brief statement of the differences in the ways in which you would present science to a child of about ten, to one of 14, and to one of 18 or 20.

4. Have you any further criticisms or suggestions concerning the eleven theses?

5. Have you any further criticisms or suggestions about the definition of the unit as printed?

6. Would it strengthen the teaching of physics, if there were introduced a system of state certification of high school teachers similar to that for elementary teachers? If so, what would you suggest as the minimum requirement for a certificate that would entitle the holder to teach physics in the high school?

Since sending out the last circular, other associations have added representatives to the commission as follows: The New England Association of Colleges and Preparatory Schools, E. H. Hall, Harvard University, Cambridge, Mass. The Association of Colleges and Preparatory Schools of the Southern States, C. A. Perkins, University of Tennessee, Knoxville, Tenn. The Northeastern Ohio Association of Science and Mathematics Teachers, F. T. Jones, University School, Cleveland; G. R. Twiss, Central High School, Cleveland; C. H. Burr, Oberlin Academy, Oberlin. The North Dakota Association of Science and Mathematics Teachers, E. Burch, State Science School, Wahpeton; Miss D. C. Jensen, High School, Fargo; C. C. Schmidt, Superintendent of Schools, Jamestown. The New York State Science Teachers Association has increased its committee by the addition of E. W. Wetmore, State Normal College, Albany; L. E. Jenks, High School, Ogdensburg.

This circular is being sent once more to all the addresses we have. Any further documents that may be issued will be sent only to those who respond in some way to this one. Back numbers of the circulars may be had on application, in case any have been lost in the mail. Since a number have suggested that more discussion would be possible if more time were allowed for the answers, the date for their final return is set as June 1. As before they should be sent to C. R. Mann, University of Chicago.

ZOOLOGY NOTES.

A new contribution to the literature of amoeboid movements is made by O. P. Dellinger of Clark University in the *Journal of Experimental Zoology* for September, 1906. Fresh light is thrown on the perplexing question of locomotion in Amoeba by the employment of a unique though very simple method of observation, which apparently has not before suggested itself to investigators. Mr. Dellinger places difflugiae and amoebae between two long cover glasses slightly separated, and views the movements of the protozoa *from the side*. All previous conclusions seem to have been based on observations made from above.

Contrary to the results of the quite recent investigations of Professor Jennings on amoeba verrucosa, Mr. Dellinger finds that amoebæ do not progress by a rotation of the protoplasm. Not even the appearance of such a rotation could be seen in species other than verrucosa when observed in the usual way from above. Mr. Dellinger employed Professor Jennings' method of mixing soot with the water in which the amoebæ were moving. Particles of this soot become attached to the surfaces of the organisms, and reveal what currents are present in them. When viewed from the side, the writer asserts that Amoeba verrucosa, Amoeba proteus, and several other species, as well as Difflugia acuminata and Difflugia spiralis, perform locomotor movements in the following manner: A pseudopod is extended free in the water; this becomes attached near its tip; there then take place a contraction of the protoplasm back of the point of attachment, and a flow of substance toward the tip of the pseudopod. The movements appear to be due to the presence of a contractile substance in the protoplasm. The granules of the endosarc do not flow as if they were suspended in a fluid contained within a contractile sac, but seem to be held in definite positions under conditions which would indicate that the endosarc has a definite structure. The writer believes that a course reticulum of contractile substance distributed through the endorsarc would account for the phenomena observed.

Directions for collecting Volvox and for keeping a supply alive in the laboratory are given by Bertram G. Smith in the January number of the *American Naturalist*. In the vicinity of Ann Arbor the writer finds two species of Volvox. In the early spring Volvox globator (the form described in text-books), and no other species, is found abundantly in small glacial pools containing Riccia and duckweed. In late autumn only Volvox aureus occurs in the same pools. During the summer both species may be found together.

The following key for the determination of the two species is given by Kofoid:

Cells about 10,000 (minimum 1,500, maximum 22,000), angular with stout connecting protoplasmic processes into which the chromatophore may enter. Diameter of colony about 700 micra (minimum 400, maximum 1,200); diameter of cell body 3—5 micra. *Volvox globator* LINN.

Cells 500—1,000 (minimum 200, maximum 4,400); rounded, with slender connecting protoplasmic processes into which the chromatophore does not enter. Diameter of colony 170—180 micra; diameter of a cell body 5—8 micra. *Volvox aureus* EHRENN.

Volvox globator is recommended as the better form for laboratory work, and when this species is to be studied in the fall it is suggested that material be gathered in the spring, and preserved until needed, in four per cent formalin.

In the laboratory Volvox should be kept in the same water in which it was found, not in tap water. Mr. Smith uses for aquaria shallow glass dishes, covered with glass plates, and placed near windows. In the water with the Volvox he keeps a moderate amount of decaying plant or animal material, from which the organisms probably derive carbon dioxide. The temperature of the aquaria is not allowed to rise above that of the ponds in which Volvox lives. This is prevented by partially immersing the aquaria in running tap water. Under these conditions Volvox has been kept alive for several weeks. Usually after one or two weeks in the laboratory the organisms reach the sexual stage, in which they become less motile, and often drop to the bottom of the dish, sometimes becoming concealed in the ooze which settles there.

In the same number of the *Naturalist* Professor F. C. Waite points out that the correct specific name of Necturus is *maculosus*. This term was originally applied to the animal in 1818 by Rafinesque. In a detailed review of the systematic literature dealing with Necturus the writer shows the priority of *maculosus* over *maculatus*, *lateralis*, and other specific names.

MATHEMATICAL CORRECTION.

On page 147 of the February number (1907) a writer, speaking of a ball which traces a sinuous path down a groove of circular section states that the period of vibration is got from the formula for an equivalent simple pendulum, its length to be $\frac{7}{5}(R-r)$. This is a slip which it may be worth while to correct. The length of the equivalent simple pendulum is

$$\left[(R-r) + \frac{2}{5}\frac{R^2}{R-r}\right]\sec i$$

where R is the radius of the groove, r of the ball, and i the inclination of the plane. If we denote the distance down the incline by x, the angle of the groove the ball has mounted from its center Θ, we find that the potential energy of the ball is

$$U = mg\,[(R-r)(1-\cos\Theta)\cos i + x \sin i]$$

and its kinetic energy

$$T = \frac{1}{2}m\left(\frac{d\Theta}{dt}\right)^2\left([R-r]^2 + \frac{2}{5}R^2\right) + \frac{7}{10}m\left(\frac{dx}{dt}\right)^2$$

If then we form the Lagrangian equations of motion from $L=T-U$ we get

$$\frac{d^2x}{dt^2} = \frac{5}{7}g\sin i$$

$$\frac{d^2\Theta}{dt^2} = g\sin\Theta\,\frac{(R-r)\cos i}{(R-r)^2 + \frac{2}{5}R^2}$$

WILLIAM R. RANSOM, Tufts College, Mass.

REPORT OF MEETINGS.

ASSOCIATION OF TEACHERS OF MATHEMATICS IN THE MIDDLE STATES AND MARYLAND.

New York Section.

On Thursday evening, February 7, 1907, the New York section of the Association of Teachers of Mathematics of the Middle States and Maryland held a dinner at Reisenweber's Hotel, Fifty-eighth street and Eighth avenue, at 6:30. After the dinner the subject of "Opportunities for Mathematicians" was discussed: 1, "In Engineering Work." Addams Stratton McAllister, M.M.E., Ph.D., associate editor of the Electrical World and Engineer; 2, "Actuarial Life Insurance," Wendell G. Strong, Ph.D., assistant actuary o fthe Mutual Life Insurance Company; 3, "In Colleges, Normal Schools and Secondary Schools," Professor David Eugene Smith, Columbia University.

Rochester Section.

A meeting of the Rochester section of the Association of Teachers of Mathematics in the Middle States and Maryland was held at the University of Rochester, February 23. The meetings were in the Physics lecture room of the Eastman laboratories. The following program was carried out:

Morning Session.—Short addresses of welcome: President Rush Rhees, the University of Rochester; Principal A. H. Wilcox, East High School, Rochester; Principal H. S. Weet, West High School, Rochester. The Literature of Secondary Mathematics: Professor A. S. Gale, the University of Rochester. Appointment of Committee on Organization.

Afternoon Session.—Report of Committee on Organization and Election of Officers, Recent Tendencies and Open Questions in the Teaching of Elementary Geometry, Mr. W. Betz, East High School, Rochester. How the Study of Physics helps the Teacher of Mathematics, Professor H. D. Minchin, The University of Rochester.

Chemistry Teachers' Club of New York City.

The second meeting of the Chemistry Teachers' Club for this school year took the form of an excursion to the Bradley Plant of the National Lead Company. This trip of Saturday, November 17, 1906, was in the nature of a supplement to that of October 27, on which date the Atlantic White Lead Works of the same company were visited.

The output of the Bradley works consists of:

1. Litharge by oxidation of lead in air.
2. Litharge by oxidation of lead by sodium nitrate.
3. Red lead by oxidation of litharge in air.
4. Orange mineral from white lead.
5. Sugar of lead from metallic lead.
6. French process white lead as an intermediate product.
7. Sodium nitrite as a by-product.

The number and length of the processes observed prohibit a detailed statement of what was seen.

For the courtesy shown by the company's representatives both on this and on the preceding visit, the club wishes to express its appreciation.

Jos. S. Mills, Secretary.

Eighteenth Meeting.

After the usual informal dinner at the Hotel Savoy, the members of the Chemistry Teachers' Club assembled for the eighteenth regular meeting in the chemistry lecture room of the new De Witt Clinton High School, at 8:20 P. M., Saturday, December 8, 1906. The opening of the meeting, however, was delayed for about fifteen minutes to give the speakers of the evening time to set up apparatus.

Following the reading of the minutes came a report from the Auditing Committee and the appointment of two standing and one short-time committee, viz., a Journal, a Printing and a (laboratory) Balance Committee, the duty of the last named being to find or devise a cheap and sensitive balance for beginners' "quantitative" experiments.

After the election of eight new members and the reading of two communications, the regular program of the evening was taken up. The number of topics considered would render even a short review of them too lengthy for this article. It is hoped that number of them may be published later in the School Science and Mathematics. The program was as follows:

1. Wadleigh High School............................J. S. Gibson
   Blowpipe Analysis using Plaster Tablets.
   Contact Sulphuric Acid.
2. Pratt Institute....................................C. M. Allen
   Quantitive Estimation of Carbon Dioxide in Calcite.
3. New Haven High School......................B. F. McFarland
   A Simple Voltameter.
   A Bunsen Burner Igniter for Laboratory Use.
4. Erasmus Hall High School—
   Illustrations of Faraday's Law...........E. R. VonNardroff
   Electrolysis of Hydrochloric Acid............W. J. Hancock
5. Stuyvesant High School.........................R. W. Fuller
   A Laboratory Device for Chlorine Experiments.
   Inorganic Preparation Charts.
6. Teachers' College...............................J. F. Woodhull
   Apparatus for Generating Hydrogen by Sodium, etc.
7. Hasbrouck Institute...............................P. C. Hyde
   Modified Kipp Generator.
8. De Witt Clinton High School....................J. E. Whitsit
   Simplified Vapor Density Apparatus.
9. High School of Commerce—
   Qualitative Reduction by Hydrogen............C. H. Wagner
   Catalytic Sulphuric Acid..........................J. S. Mills

At 10:50 P. M. the meeting adjourned for a brief inspection of the new De Witt Clinton High School chemical laboratories.

Jos. S. Mills, Secretary.

MISSOURI SOCIETY OF TEACHERS OF MATHEMATICS AND SCIENCE.

The fourth regular meeting of the Missouri Society of Teachers of Mathematics and Science, was held at Moberly, December 27 and 28, in connection with the State Teachers' Association. Thursday afternoon was given to a joint meeting of the two branches of the Society. Chester B. Curtis, of St. Louis, presented an interesting paper on "The Genesis of Cosmic Systems." He reviewed the various theories or hypotheses that have been advanced to account for the planetary system as well as for other similar systems. The oldest known is that of La Place, whose hypothesis seems to be confirmed by the fact which has been proven that the sun is contracting regularly at the rate of about forty miles per century.

Herschel's work with nebulæ was reviewed, and following this the study that Darwin has given in recent years to the tides. He has shown that the length of our day has been gradually increasing and at the same time the distance of the moon from the earth. If we go back for an indefinite number of centuries this would bring us to a time when the day was but a few hours in length and the moon very close to us. Going back still further we should find the moon a part of the earth itself.

T. J. J. See, of the Naval Academy, has advanced another explanation for the heat of the sun. He calculates that the temperature of nebulæ is about 272 degrees Centigrade, that is, only one degree above zero absolute. When in the nebulous condition the sun undoubtedly extended out beyond the most distant of the planets. Knowing that the contraction of a gas is accompanied by an increase in temperature, See assumes that as this nebulous mass has contracted it has constantly risen in temperature till that of the sun now is not far from eight thousand degrees Centigrade.

Chamberlain, of Chicago University, has advanced another theory for the formation of worlds. He believes that the vast protuberances which the sun projects from its surface from time to time, have often been so great that the mass thrown outward has come within the range of attraction of other suns; that in this way the mass has become detached and has then begun a separate existence.

Friday afternoon the Science section had for consideration, "The Use of the Lantern in Science Teaching." This was discussed from four viewpoints: Physics, Chemistry, Physiography and Biology.

F. H. Ayres, of Kansas City, presented the first. He gave a large number of experiments with the lantern, showing how he used it with his classes. Among these were: first, several to illustrate surface tension —the weakening by means of a drop of alcohol upon water of the tension between two floating toothpicks, a camel's-hair brush wet and dry, the camphor boat, the wire-gauze boat; the study of the action of a battery, the galvanometer, induction, the electroscope, and a number of experiments in color work. These experiments were greatly enjoyed by the science teachers present. Mr. Ayres also showed how to prepare

a vertical projection apparatus at almost no expense, and one for color work at a cost of $1.50.

The work in chemistry was given by W. A. Lewis, of the Kirksville Normal. By the use of quite a number of well-chosen slides he showed how various phases of general chemistry can be made more profitable and much more interesting. Some of these illustrated methods of mining, others the reduction of ores, and others metallurgical processes, manufactory, etc. He also showed here the study of various crystalline forms that the chemical student may be shown on the screen.

N. W. Fuller of St. Louis, had the work in physiography. The lantern in physiographical work comes under three general classes: introductory, explanatory and illustrative. The majority of high-school students take up physiography before physics. The lantern therefore is of much value in introducing the ideas of coloration, spectrum, refraction, etc., before the work on rainbows, sunset colors, clouds, etc. From the explanatory standpoint, such a subject as solar and lunar eclipses may be worked out with the lantern and two globes before the class. It offers a means to illustrate features and conditions which are not local. It places the same thing before the class as a whole and thus offers a means for general discussion. It is a saving both of time and energy, enabling the teacher to handle the subject with the class as a unit and not individually.

J. W. Cameron, of Kansas City, gave a large number of slides, illustrating the study of plant and animal life. Many of these were photographs of birds in their native haunts.

The next subject for discussion was the preparation of slides. No formal paper was offered, but a number of teachers present gave their methods of making slides.

In the Mathematics Division Mr. A. J. Schwartz of the McKinley High School, St. Louis, Mo., spoke on "The Rational Treatment of the Subject Matter of Plane Geometry." He said that the current text-books on Plane Geometry do not discriminate sufficiently between essentials and non-essentials. The current text-books are deluged with a great number of stock propositions, a large proportion of which are unimportant theorems. He takes the ground that the propositions should be reduced to essentials, and that they should be selected in accordance with their usefulness in the development of important geometric relations. He made the point that where this selection is not made, that is, where important and unimportant theorems are emphasized in the same degree, the pupils acquire a distaste for the subject; they fail to see it in proper perspective; moreover, pupils so taught are at a disadvantage when they attempt to apply their knowledge, for useless principles are as likely to appear in the foreground of consciousness as the principles they need.

An important problem for the teacher of geometry is to select from the maze of the subject matter the few necessary basic principles, and intensify and reinforce them until they become integral parts of the pupils' mental equipment—ready for use at any time. To illustrate how this may be done, he gave a detailed outline of a particular topic,

namely, "Similar Polygons." For this topic he selected as basic principles the following theorems: "Two triangles are similar if they have two angles of the one equal to two angles of the other; Two triangles are similar if they have an angle of the one equal to an angle of the other and the including sides proportional; Two triangles are similar if they have their sides respectively proportional."

These basic principles were followed by a list of minor theorems and problems, including such theorems as: "If two triangles have their sides respectively parallel or respectively perpendicular, they are similar; If two chords of a circle intersect, the ratio of either segment of the first to either segment of the second is equal to the ratio of the remaining segment of the second to the remaining segment of the first; The corresponding altitudes of two similar triangles have the same ratio as any two homologous sides; If two polygons are similar, they may be decomposed into the same number of similar triangles," etc. These theorems were graded so as to encourage independent work.

The chief function of these accessory theorems and problems is to give a thorough grasp of the basic principles, so that they may be readily recalled to the foreground of consciousness at the opportune time and intelligently applied.

ARTICLES IN CURRENT MAGAZINES.

American Naturalist for January: "Note on the Habits of Fierasfer," Professor Edwin Linton; "Records of Pennsylvania Fishes," Henry W. Fowler; "Specific Name of Necturus maculosus," Professor F. C. Waite; "Volvox for Laboratory use," Bertram G. Smith; "Ostracoda from Southeastern Massachusetts," J. A. Cushman; "Notes and Literature, Physics, A First Course in Physics; Biology," Jennings'. For February: "An Automatic Aerating Device for Aquaria," Dr. Louis Murbach; "The Flying-Fish Problem," Lieut.-Col. C. D. Durnford; "Cataloguing Museum Specimens," Professor L. B. Walton; "Some South American Rotifers," James Murray; "Meristic Homologies in Vertebrates," J. S. Kingsley; "On the Osteology of the Tubinares," Dr. R. W. Shufeldt.

Forestry and Irrigation for February: "Agricultural Settlements in Forest Reserves;" "Economy in Railroad Uses of Wood," Wm. L. Hall; "Financial Results of Forest Management," Bernhard Eduard Fernow (Illustrated); "Demand for the Passage of the Appalachian White Mountain Bill," The South in Earnest; "Notes on Robinia Neo-Mexicana," Frank J. Phillips (Illustrated); "How Fast are We Cutting Timber?" George K. Smith; "Progress of Forestry Education," Henry S. Graves.

Good Housekeeping, February: "Mushrooms," J. H. Adams; "Pure Food Assurance."

The Literary Digest, February 2: "After-shocks of the Kingston Earthquake;" "Our Defenseless Coasts and Harbors;" "Is Machinery a Curse?" "Assimilation, Benevolent and Otherwise." March 2, "Evils of Boiled Water," "Steam Power from the Earth's Internal Heat," "A New 'Wick' for Arc Lights."

Monthy Weather Review, October: "Suggestions as to Teaching the Science of the Weather," J. W. Smith; "Has the Gulf Stream any Influence on the Weather of New York City?" James Page; "On the Formation of Anchor Ice, or Ground Ice, at the Bottom of Running Water," H. T. Barnes. November: "Lunar Rainbow at Tampa, Fla.," J. S. Hazen; "The Origin of our Cold Waves," C. A.; "The Evaporation of Ice," F. C. Mitchell.

Nature-Study Review for January: "Established Principles of Nature-Study," M. A. Bigelow; "Established Principles, Discussion," C. F. Hodge; "Field Work in Botany," C. E. Bessey.

Ores and Metals for February: "Colorado Mining Demands Restoration of Natural Protection;" "Discovery of Key to Greatest Unutilized Mineral Treasure;" "Preventable Waste in Destruction of Human Life;" "Experience With Nitro-glycerine," Thomas Withers; "New Mexico Mining Progress in 1906," Professor R. V. Smith; "Tunneling Machine for Cripple Creek Project—State Geological Survey;" "Cement Demand as Indicator of Prosperity in South Africa;" "Geological Surveys Rely on Universities," R. D. George; "Platinum Minerals and Where They Are Found;" "Timber Used in United States Mines in 1905," R. S. Kellogg; "Magmatic Quartz as an Ore Considered in J. E. Spurr's Report on the Silver Peak Quadrangle."

Photo Era, January: "A Factorial Development Chart;" "The Photography of Snow Landscapes;" "Development Without Rocking;" "Mars and Recent Researches." February: "Photography in Jamaica;" "Notes on Flashlight Portraiture;" "Mars and Recent Researches."

Physical Review for February: "The Latent Heat of Recalescence in Iron and Steel," F. K. Bailey; "Experiments on Resonance in Wireless Telegraph Circuits," Part V, George W. Pierce; "The Transformation into an Electric Current of Radiation Incident on a Moving Surface," Bergen Davis; "The Elastic Modulus for Small Loads at the Elastic Limit," Henry W. Bearce; "Index of Refraction and Dispersion with the Interferometer," C. A. Proctor; "The Luminous Equivalent of Radiation," P. G. Nutting; "Standard Cells," K. E. Guthe and C. L. Von Ende; "An Optical Device for Deflection Instruments," Edwin F. Northrup; "Theory of the Electrodeless Ring Discharge," Bergen Davis; "American Physical Society," Minutes of the Thirty-fifth Meeting.

Popular Science Monthly for February: "Glacial Erosion in Alaska," Professor Ralph S. Tarr; "The Relation of School Organization to Instruction," Professor Wilbur S. Jackman; "In Search of Truth," President David Starr Jordan; "Is Man an Automaton?" Professor George Stuart Fullerton; "A Vocabulary Test," Professor E. A. Kirkpatrick; "Magical Medical Practice in South Carolina," John Hawkins; "The Value of Science," M. H. Poincaré. For March: "The Century Plant and Some Other Plants of the Dry Country," Professor William Trelease; "Notes on the Development of Telephone Service," Fred DeLand; "Denatured Alcohol," Professor S. Lawrence Bigelow; "Spelling Reform and the Conservation of Energy," Professor W. Le Conte Stevens; "The Value of Science," M. H. Poincaré.

Review of Reviews for February: "Manufacturing in South America," G. M. L. Brown and Franklin Adams; "Protecting the Farmer Against Fraud," John Phillips Street; "The Secret of Successful Motoring," M. C. Krarup. For March: "The Interior Department Under Secretary Hitchcock," Max West, with portraits; "Guarding the Public Coal Lands;" "The Jamestown Exposition," Plummer F. Jones, with portraits and other illustrations; "The Municipal Ownership of Street Railroads in Germany," Edward T. Heyn; "German Land Tax Experiments," William C. Dreher; "Why Not Savings-Bank Life Insurance for Wage-Earners?" Louis D. Brandeis.

School Review for January: "Should High-School Botany and Zoölogy Be Taught with Reference to College Entrance Requirements?" Otis W. Caldwell; "On the Teaching of Secondary Mathematics," John J. Schobinger.

School World for February: "Measurement of Time-Intervals in Experimental Mechanics," J. Schofield.

Science, February 1: "The Contributions of America to Geology," William N. Rice.

Scientific American, February 2: "Cobalt Mining in Canada," Allen Porter; "Concerning Ears," G. H. Lydekker. January 26: "A New Electric Lamb Filament."

Scientific American Supplement, January 12: "Scientific Aspects of Luther Burbank's Work." March 2: "New Way of Making Phosphorus;" "A Study of Color Phenomena;" "Manufacture of Illuminating Gas;" "Black Sand Investigations."

Technical World, for February: "Ship Canal Across Cape Cod;" "Putting the Mountains to Work;" "Tiny Master of Vast Engines;" "Wiring the Wilderness;" "City Rubbish Turned to Light;" "Rooms That Heat Themselves;" "Soda from Dry Lakes." For March: "Dwarfs Niagara's

Power;" "How Jackson Saved the Eskimo;" "Rediscovered Texas;" "Weird Monsters of the Sea;" "Wireless Control of Mechanisms;" "How Good is Concrete?" "In the Track of the Hurricane."

Zeitschritt für den physikalischen und chemischen Unterricht, Januar, 1907: E. Grimsehl, "Uber den Hochschulunterricht für künftige Lehrer der Physik;" E. Grimsehl, "Ein Apparat zum Messen der Zusammendrückbarkeit des Wassers;" E. Grimsehl, "Ein Apparat für Magnetinduktion;" Fr. C. G. Müller, "Neue Versuchsanordnung zur Synthese des Chlorwasserstoffs und des Wassers;" K. Schreber, Das 'funktionale Denken' im Physikunterricht;" W. Bahrdt, "Enige Schulversuche zur Ausdehnung von Gasen durch die Wärme;" W. Volkmann, "Ein objecktiver Beugungsversuch zur Abbeschen Theorie des Mikroskopes."

BOOKS RECEIVED.

Report of the Botanical Club of Canada for 1905-1906, by A. H. MacKay.

Bibliography of Canadian Botany, by A. H. MacKay.

The Educational Significance of Sixteenth Century Arithmetic from the point of view of the present time, by L. L. Jackson, head of the department of mathematics, State Normal School, Brockport, N. Y. Pp. 232. Published by Teachers' College, Columbia University, 1906.

The Teaching of Mathematics in the Elementary and the Secondary School, by J. W. A. Young, assistant professor of the pedagogy of mathematics in the University of Chicago. Pp. 351. Longmans, Green & Co., New York, 1907. Price, $1.50.

Animal Micrology: Practical Exercises in Microscopical Methods, by Michael F. Guyer, Ph.D., professor of zoology, University of Cincinnati. Pp. 240. University of Chicago Press, 1906. Price, post paid, 1.88.

BOOK REVIEWS.

Animal Micrology. M. F. Guyer, Ph.D. 8vo; 240 pages; 71 figures. Net, $1.75. The University of Chicago Press.

A new book on microscopical technique, prepared by an experienced teacher, and certain to be very useful to the teacher of zoology who is not accustomed to the ordinary technique of preservation and preparation of animals for microscopical study, and also to be very convenient for reference for those of more experience. The author has been quite successful in accomplishing his purposes as outlined in the preface. "The aim of the entire book is to be practical; to omit everything that is not essential; and, above all, to give definite statements about things. The book is intended primarily for the beginner and gives more attention to the details of procedure than to discriminations between reagents or to the review of special processes. The student is told what to do with his material step by step, and why he does it; at what stages he is likely to encounter difficulties and how to avoid them. A very brief, non-technical account of the principles of the microscope is inserted (Appendix A) with the idea of giving the student just enough of the theoretical side of microscopy to enable him to get satisfactory results from his microscope. In Appendix B the formulæ for a number of the most widely-used reagents are given with comments upon their uses

and manipulation. Following this (Appendix C) is a concise table of a large number of tissues and organs with directions for properly preparing them for microscopical study. In Appendix D some directions are given for collecting and preserving material for an elementary course in zoology."

FRANK SMITH.

Consul-General Frank H. Mason makes a report from Paris on a new process for producing hydrogen, as follows:

At a recent meeting of the French Academy the eminent physicist, Mr. Moissan, presented a report from Mr. Georges F. Joubert describing a new and thus far secret process for the manufacture of hydrate of calcium, a product which, by reason of its convenient fertility for the generating of hydrogen gas for ballooning and other purposes, is likely to play an important role in the field of applied chemistry.

It appears that the Société d'Electrochemie, at St. Michel de Marienne, has succeeded, like the Electro-technical Company, at Bitterfield, Germany, in producing by electrical process calcium metal on a commercial scale and at a price so moderate as to permit its use for various industrial purposes.

The invention of Mr. Joubert consists in a process by which the reaction of metallic calcium upon a metallic salt produces the new form of hydrate of calcium, or, as it is commercially known, "hydrolithe." This resembles in appearance and qualities calcium carbide, with the difference that whereas the carbide with the addition of water evolves acetylene gas, the hydrolithe upon contact with water evolves hydrogen gas. When pure, one pound of hydrolithe will generate 18.46 cubic feet of hydrogen. When of the ordinary commercial grade of purity, one pound of hydrolithe will create 16.05 cubic feet of gas.

Its most ready and obvious use is thus far for inflating balloons for military and other purposes. It is safe and easy to handle, can be used for generating gas wherever water can be obtained, and for long flights can be carried as ballast instead of sand, and employed at will for refilling the balloon, which may thus be kept in flight almost indefinitely. As an illustration of the economy of weight that has been accomplished by the substitution of hydrolithe for the purposes of military balloon service, it may be stated that an ordinary field balloon contains, when inflated, about seventeen thousand six hundred and fifty-seven cubic feet of gas, the generation of which by the means hitherto employed requires the employment of materials and apparatus which fill three wagons, each one of which weighs when loaded three and a half tons, and requires in a campaign to be drawn by six horses. All this cumbrous and costly equipment can now be replaced by a two-horse wagon carrying a ton of "hydrolithe," which, with the addition of water that can be obtained anywhere, supplies instantly and in controllable quantities whatever gas may be required.—*Scientific American.*

ERRATUM.

On page 236 of the March issue calsium should read Caesium.